Exam 2 Algebra & Trig 3/19/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Write an equation for the line through the point (-3,2) with a slope of 2/3.

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{2}{3}(x+3)$

$y = \frac{2}{3}x + 4$

Good

2. What are the slope, x-intercept, and y-intercept of the line with equation $-3x + 2y = 15$?

$-3x + 2y = 15$

$2y = 3x + 15$

$y = \frac{3}{2}x + \frac{15}{2}$

$3x = -15$

$x = -5$

Slope $= 3/2$

y-intercept $= 15/2$

x-intercept $= -5$

Great

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. What are the center and radius of the circle with equation $x^2 + y^2 - 4x + 12y = -36$?

$x^2 - 4x + 4 + y^2 + 12y + 36 = -36 + 36 + 4$

$(x-2)^2 + (y+6)^2 = 4$

radius $= \sqrt{4} = 2$

$\boxed{\text{radius} = 2 \quad \text{Center} = (2, -6)}$

Nice Job

$(x-1)$

4. Factor the function $f(x) = 2x^3 - 7x^2 - 7x + 30$ completely.

- Possible Roots

$30 \to 1, 2, 3, 5, 6, 10, 15, 30 \to \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$2 \to 1, 2$ $\quad \pm 1/2, \pm 3/2, \pm 5/2, \pm 15/2$

- Zeros at $-2, 5/2, 3$

$\boxed{f(x) = (x+2)(2x-5)(x-3)}$

Well done

$(x+2)(2x-5) \to 2x^2 - 5x + 4x - 10$

$(x-3)(2x^2 - 5x + 4x - 10)$

$2x^3 - 5x^2 + 4x^2 - 10x - 6x^2 + 15x - 12x + 30$

$2x^3 - 7x^2 - 7x + 30$

5. If $f(x) = x^2$, $g(x) = x + 5$, and $p(x) = 5x^3 - 2x$, find:

(a) $f(3) = 3^2 = 9$

(b) $f \circ g(-1) = f(4) = 4^2 = 16$

(c) $p(x + h) = 5(x+h)^3 - 2(x+h)$

Great

6. Write one possible formula for a rational function with vertical asymptotes at $x = -4$ and $x = 5$, a horizontal asymptote at $y = 3$, and an x-intercept at $x = 3$.

$$p(x) = \frac{3\cdot(x-3)^2}{(x+4)(x-5)}$$

To have a horiz. asymp. at $x=-4$, put $(x+4)$ in the denominator, and for the asymp. at $x=5$, put $(x-5)$ in the denominator. To get an x-int. at $x=3$ put $(x-3)$ in the numerator.

Then for the horiz. asymp., the degrees of numerator and denominator have to be equal, so just make it $(x-3)^2$ in the numerator. Then put a 3 on top to make the highest term there $3x^2$, compared to x^2 on the bottom.

10. Find all real solutions to the equation $x + \sqrt{6x} = 2x^2$.

Isolate Radical: $\sqrt{6x} = 2x^2 - x$

Square: $6x = 4x^4 - 4x^3 + x^2$

Rearrange: $0 = 4x^4 - 4x^3 + x^2 - 6x$

From a calculator graph, this graph crosses the x-axis at $x=0$ and $x = 3/2$.

$x = 0$

$x = 3/2$

7. Find all roots, real and complex, of the polynomial $3x^4 + x^3 + 4x^2 - 4x$.

$4 \rightarrow 1, 2, 4$ $\qquad$ $\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

$3 \rightarrow 1, 3$

$x(3x^3 + x^2 + 4x - 4)$

$\boxed{\underline{0}, +\underline{2/3}} \rightarrow$ from Calculator

$x(3x-2) = 3x^2 - 2x$

$$
\begin{array}{r}
x^2 + x + 2 \\
3x^2-2x \overline{\big) 3x^4 + x^3 + 4x^2 - 4x} \\
\underline{-3x^4 + 2x^3 \qquad\qquad} \\
3x^3 + 4x^2 - 4x \\
\underline{-3x^3 + 2x^2 \qquad} \\
6x^2 - 4x \\
\underline{-6x^2 + 4x} \\
0
\end{array}
$$

Excellent!

$$\frac{-1 \pm \sqrt{1 - 4(1)(2)}}{2} = \frac{-1 \pm i\sqrt{7}}{2}$$

$$\boxed{0, +\frac{2}{3}, \frac{-1 \pm i\sqrt{7}}{2}}$$

8. Decompose $\frac{3x+31}{x^2+2x-15}$ into partial fractions.

$$\frac{3x+31}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

$$A(x-3) + B(x+5)$$

$$3x+31 = Ax - 3A + Bx + 5B$$

If $x = 3$:

$40 = 8B$

$\therefore B = 5$

If $x = -5$:

$16 = -8A$

$\therefore A = -2$

So $\frac{3x+31}{x^2+2x-15} = \frac{-2}{x+5} + \frac{5}{x-3}$

One Method

Match coef. of x:

$3 = A + B$

$3 - B = A \longrightarrow$

Match coef. of const.:

$31 = -3A + 5B$

$31 = -3(3-B) + 5B$

$31 = -9 + 3B + 5B$

$40 = 8B$

$3-(5) = A \longleftarrow 5 = B$

$-2 = A$

So $\frac{3x+31}{x^2+2x-15} = \frac{-2}{x+5} + \frac{5}{x-3}$

Other Method

9. Polly is a Precalc student at Enormous State University, and she's having some trouble with functions. She says "I just so totally don't get all this stuff about graphing functions. There's all these problems where they show you this graph, and then they add or subtract or put a minus on it or something, and you're supposed to say what the new graph would be like, and I totally hate them. There was this one on our quiz, and we were supposed to move it around a couple different ways, and the grader wrote that I did it in the wrong order and gave me no points, and then the professor wouldn't help me during office hours because he said I was stupid and should already know it. I cried for like an hour!"

Explain to Polly as clearly as possible how the order in which transformations are applied to a function can affect the graph, and how to tell in which order they should be done.

Hey Polly, don't take it so hard. Here's the deal: If you take a function like $f(x) = x^2$ and compare it to $g(x) = x^2+1$, the graphs will be the same except $g(x)$ is slid up by one unit. But $h(x) = (x+1)^2$ will be like the graph of $f(x)$ slid left by one unit. So if you apply the square first and then add 1, that does one thing, but if you add 1 first and then square, it does something different.

Here's another one to think about: you know $2x^2$ is like a stretched version of x^2, always twice as far from the x axis. Now think about if you add one and then double, so $2(x^2+1) = 2x^2+2$, or on the other hand if you double and then add one, so $2x^2+1$. So in one case it ends up moved up two units, but in the other case just up one.

So Polly, the basic thing is that adding inside the function moves left, and adding outside the function moves up, so it's a matter of when you add versus when you apply the function. But there are also issues with order when you multiply, too. When you're not sure, my professor said just start from the "x" and work your way out.

Good luck when you get to the test - remember the quiz won't hurt you much if you show you got it figured out eventually!