Practice Exam 3 Answers Algebra & Trig 4/22/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

As usual, only answers are given here – remember that without justification, these would generally receive at most half credit.

1. Convert 40° to an equivalent exact radian measure.

$\frac{2\pi}{9}$

2. Find $\log_2 32$ exactly.

5

3. If θ is a fourth-quadrant angle such that cos θ = 1/5, find exact values for the other 5 trig functions of θ.

$\sin\theta = \frac{-\sqrt{24}}{5}$

$\tan\theta = -\sqrt{24}$

$\sec\theta = 5$

$\csc\theta = \frac{-5}{\sqrt{24}}$

$\cot\theta = \frac{-1}{\sqrt{24}}$

4. Sketch at least two complete periods of the graph of $y = 5\cos(6x - 9) + 10$.

The graph should begin a cosine cycle when x is $\pi/6$ at a height of 15, and complete that cycle when x is $\pi/2$ again at a height of 15. Midway in between it should hit its lowest height of 5 when $x = \pi/3$. The second cycle should be the same, ending when $x = 5\pi/6$ (unless you pick some unusual pair of cycles to plot, which is fine too).

5. Solve (exactly) the equation $\ln 3 - \ln (x + 2) = \ln x$.

$x = 1$ ($x = -3$ is an extraneous solution)

6. An office worker with a really boring job is staring out his boss's window (office workers with really boring jobs don't have windows of their own). He measures the angle made between the vertical side of his tall office building and a hotdog vendor's stand across the street to be 15°, and then at lunchtime goes out and measures that the distance between the base of the building below his boss's window and the hotdog vendor's stand is 40 feet. Find, to the nearest foot, the height of his boss's window.

149 feet

7. Simplify $\sec(\sin^{-1} x)$.

$$\frac{1}{\sqrt{1-x^2}}$$

8. Paul is a Precalc student at Enormous State University, and he's having some trouble with trigonometry. He says "I just so totally don't get all this stuff about trig functions. I figured I could pretty much just know nothing since my calculator does it all, but I totally crashed and burned on the test, and now the professor is saying since everybody else did bad too he's probably gonna put that on the final too. There was this problem with, like, the tangent of three pi over two, so I did it on my calculator, and I took 3.14 for pi, and times three and divided by 2, then I did the tangent of that. So I got, like, four hundred something, and I got totally no credit for it, and the professor was going off when he handed back the tests about how we just hit buttons and don't understand anything. But I figure the calculator's gotta be right, doesn't it? So I've got no idea what went wrong, or how I'm supposed to do it now."

Explain clearly to Paul what went wrong with his approach, and how he should deal with a problem like this.

Answers will vary, but should certainly focus on the fact that the graph of tangent has a vertical asymptote when $x = 3\pi/2$, so the real answer is undefined, but Paul's approximation is only close to that asymptote.

9. If $f(x) = 20 + 70e^{-x/50}$, find $f^{-1}(x)$.

$$f^{-1}(x) = -50 \ln\left(\frac{x-20}{70}\right)$$

10. If the population of the world is 5 billion people today, and grows at a rate of 4% per year, how long will it be until the population is 6 billion people? (Approximate your answer to 2 decimal places)

4.65 years

Extra Credit (5 points possible): A circle with a radius of 10 fits inside an equilateral triangle so that it just touches all three sides of the triangle. What is the exact area of the triangle?

$300\sqrt{3}$