

Final Exam A Algebra & Trig 5/20/2003

Each problem is worth 20 points. For full credit provide complete justification for your answers.

Remember this is a practice exam, not a study guide – the sampling of material on the real final will probably be different, and if *all* you know how to do is exactly the problems on this test you'll probably fail.

Also remember that only answers are given here, and without justification these would generally receive at most half credit.

1. Find the exact distance between the points (4,-2) and (6,3).

$$\sqrt{29}$$

2. Write an equation for the line passing through the points (4,-2) and (6,3).

$y + 2 = \frac{5}{2}(x - 4)$ and $y = \frac{5}{2}x - 12$ are among the possible ways it could be written.

3. Find exact values for all solutions (both real and complex) to the equation $x^2 + 5x = 3$.

$$x = \frac{-5 \pm \sqrt{37}}{2}$$

4. If $\mathbf{v} = \langle -3, 4 \rangle$ is a vector, find

- (a) The magnitude of \mathbf{v}
- (b) A unit vector in the direction of \mathbf{v} .

(a) $|\mathbf{v}| = 5$

(b) $\mathbf{u} = \langle -3/5, 4/5 \rangle$

5. Convert the point with polar coordinates $(10, 2\pi/3)$ to rectangular coordinates.

$$(-5, 5\sqrt{3})$$

6. Jon's cat Nemo is planning to attack the neighbor's yappy little dog. Nemo will wait until the dog is three feet from the base of the wall beneath the porch window, then instantly shred the screen and pounce on the dog. If the window is five feet above ground level, and Nemo's trajectory once he jumps will essentially be a straight line because he's moving so fast, what angle will Nemo's path make with the vertical wall of the porch?

$$\tan^{-1}\left(\frac{3}{5}\right) \approx 30.964^\circ$$

7. (a) Find the inverse of the function $f(x) = 5(x+2)^3 - 1$.

(b) Explain clearly, as if to another student who's having trouble, what the inverse function and $f(x)$ have to do with each other.

$$(a) f^{-1}(x) = \sqrt[3]{\frac{x+1}{5}} - 2$$

(b) As always, there are lots of good ways of explaining this, but most would focus on the fact that each of these functions would “undo” what the other one does.

8. Verify the trig identity $\frac{\sec \theta}{\cot \theta + \tan \theta} = \sin \theta$.

One possible way:

$$\begin{aligned} & \frac{\frac{1}{\cos \theta}}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}} = \\ & \frac{\frac{1}{\cos \theta}}{\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}} = \\ & \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta \cos \theta}} = \\ & \frac{1}{\cos \theta} \cdot \frac{\sin \theta \cos \theta}{1} = \end{aligned}$$

9. Find all roots of the polynomial $f(x) = x^3 + 2x^2 - 3x - 6$.

$$(x+2)(x-3)(x+3)$$

10. If the temperature after t minutes of a cup of coffee is given by $T(t) = 70 + 120e^{-0.06t}$, how long will it be until the coffee has cooled to 120 degrees?

14.59 minutes, to the nearest hundredth of a minute.

Extra Credit (5 points possible): A circle is drawn around an equilateral triangle with sides of length 1 so that the circle just touches each corner of the triangle. What is the area of the circle?

$\pi/3$