Exam 1 Calc 2 2/26/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Integrate** $\int x^3 \ln x\,dx$. [Hint: the answer is $\frac{x^4 \ln(x)}{4} - \frac{x^4}{16} + C$]

2. If the work required to stretch a spring 1 foot beyond its natural length is 15 ft-lb, how much work is needed to stretch it 6 inches beyond its natural length?

3. Compute the average value of $f(x) = \sin x$ on the interval from $x = 0$ to $x = \pi$.

4. **Set up** an integral for the volume of the solid generated by rotating the region between $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$ around the y axis.

5. An aquarium 2 meters long, 1 meter wide, and 1 meter deep is full of water. **Set up** an integral for the amount of work needed to pump half the water out of the aquarium (the density of water is 1000 kg/meter^3).

6. Integrate $\int \sin^n x \cos^3 x \, dx$. [Hint: the answer is $\dfrac{\sin^{n+1} x}{n+1} - \dfrac{\sin^{n+3}}{n+3} + C$]

7. The graph of $x^2 - y^2 = 1$ is a hyperbola. Set up an integral for the area in the first quadrant bounded by the hyperbola and the line $y = 1$ and use it to find the area.

8. Because of a stunningly negligent editor named Brian, the first printing of the second edition of CliffsQuickReview Calculus was released with a table of integrals that said $\int \frac{dx}{a^2 + x^2} = \frac{1}{2} \arctan \frac{x}{2} + C$. Explain, in simple enough terms that Brian can follow along (Brian claimed to have taken calculus himself), exactly how this formula is or is not acceptable and why.

9. Suppose a quadrilateral has vertices at $(0,0)$, $(1,0)$, $(0,1)$, and a point (a,b) for which $0 < a < 1$ and $b>0$. **Set up** an integral (or integrals) for the area of this quadrilateral.

10. Jon has a bowl-shaped fountain on his desk which is shaped like a frustum of a sphere with a radius of 9 inches, cut off 3 inches up from the bottom. Set up an integral and use it to find the volume of water contained in this fountain.

Extra Credit (5 points possible): If the region under $y = [\![x]\!]$ but above the x axis, between $x = 0$ and $x = m$ (for some integer m), is rotated around the y axis, what can you say about the volume of the resulting solid?