Exam 2a Calc 2 3/19/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Write an integral** for the length of the curve y = sin x between (0,0) and (π,0).

2. **Show** that $\int_1^{\infty} \frac{1}{x^2}\, dx = 1$.

3. **Integrate** $\int \sin^{-1}\left(\frac{x}{2}\right)dx$ [Hint: Line 87 of our Table of Integrals says $\int \sin^{-1} u\, du = u \sin^{-1} u + \sqrt{1-u^2} + C$].

4. If the function f(t) = $0.5e^{-t/2}$ is the probability density function for the chances of a certain 3-month old child messing his diaper t minutes after having it changed, **what is the probability** of the child messing his diaper in the first 15 minutes after being changed?

5. Use line 21 on our Table of Integrals, which says

$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right) + C$, to show that

$$\int \sqrt{x^2+6x+10}\,dx = \frac{\ln\left(\sqrt{x^2+6x+10}+x+3\right)}{2} + \frac{(x+3)\cdot\sqrt{x^2+6x+10}}{2} + C .$$

6. **Evaluate** $\int \frac{x^3+1}{x^2+1}dx$ [Hint: The answer is $\frac{-\ln\left(x^2+1\right)}{2}+\tan^{-1}x+\frac{x^2}{2}+C$].

7. **Find** $\bar{x}$, the x coordinate of the centroid of a rectangle with vertices at (0,0), (a,0), (0,b), and (a,b).

8. Brandi is a Calculus student at E.S.U. who's having some trouble with arc length. Brandi says "Okay, so there's this formula for arc length, and I'm fine with that, it's pretty simple. But I don't think it can actually be totally right, either, because think about if it was an improper integral, like from 1 to infinity, right? So the thing is, some improper integrals converge, so they're finite. But the length of a curve that goes from where x is 1 on out forever just couldn't be finite, right? So doesn't that have to mean that the formula for arc length should, like, say that it doesn't work with improper integrals?"

Explain to Brandi either how the formula for arc length does handle this situation, or why you know for certain that it does not.

9. **Derive** the formula $\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2}+C$.

10. **Write an integral** for the surface area of a sphere with radius r.

Extra Credit (5 points possible): Show that the surface area of a zone fo a sphere that lies between two parallel planes is $S = 2\pi rh$, where r is the radius of the sphere and h is the distance between the planes.