Exam 3 Calc 2 4/18/2003

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Convert the point with polar coordinates $r = 4$, $\theta = 5\pi/6$ to rectangular coordinates.

2. Determine whether $y = e^{2t}$ is a solution to the differential equation $y'' + 3y' - 10y = 0$.

3. If a hot cup of coffee is left in a 70° F room, it is found that a general solution to the corresponding differential equation is $T(t) = Ke^{-t/30} + 70$. **Find a particular solution** representing a cup which begins at 190° F.

4. **Set up an integral** for the length of the curve with parametric equations

$$x = 2\sin 2t \qquad y = 2\sin t$$

between the point (0, 0) and the point $\left(\sqrt{3}, \sqrt{3}\right)$.

5. Let $C(t)$ be the concentration of a substance in a person's blood. If the body eliminates the substance at a rate proportional to the current concentration, then the concentration will follow the differential equation $\frac{dC}{dt} = -kC(t)$. **Find a general solution** to this differential equation.

6. **Find the exact coordinates** of the lowest point on the curve with parametric equations

$$x = 3t^3 + t,\ y = 2t^2 + t.$$

7. **Set up an integral** (or integrals) for the arc length of the portion(s) of the curve $r = \cos 3\theta$ which lies outside the curve $r = \frac{1}{2}$.

8. Beth is a calculus student at E.S.U. who wants some help with polar equations. Beth says "So I was talking about our homework assignment with this guy in my class, and there was this question about when you've got the polar equation $r = \theta$. So it's this spiral that goes around and around, and he says that it's horizontal and vertical when it crosses the axes, so like horizontal when it crosses the positive y axis, then vertical when it crosses the negative x axis, and so on. I told him it doesn't really look quite like those are the horizontal and vertical places on my calculator, but he said the calculator just distorts it a little because of the dots or rounding or something. I'm pretty sure I'm right, but I have no idea how to prove it."

Explain to Beth, in terms she can understand, either how to be sure she's right, or how to know her classmate is right.

9. **Set up an integral** (or integrals) for the area of the region between the inner and outer loops of the curve $r = 2 + 4\sin\theta$.

10. **Find the area** bounded by the parametric curve with equations $x = \cos t$, $y = \sin 2t$ [Hint: Depending on how you proceed, you might find the trig identity $\sin 2x = 2\sin x \cos x$ useful].

Extra Credit (5 points possible): Derive the formula for arc length in polar coordinates from the formula for arc length in parametric coordinates.