Exam 4 Calculus 2 5/9/2003

Each problem is worth 10 points, show all work and give adequate explanations for full credit. Please keep your work as legible as possible.

1. (a) **Write the first four terms** in the sequence $a_n = 1/(2n + 1)$.

(b) **Write the first four partial sums** of the series $\sum_{n=1}^{\infty} \frac{1}{2n+1}$.

2. **Write the Taylor polynomial** of degree 6 for $f(x) = \sin x$ centered at $a = \pi/2$.

3. **Give an example** of a series which is convergent but not absolutely convergent.

4. **Give a power series expansion** for $\dfrac{1}{1-x^5}$ in sigma notation.

5. **Show** that the radius of convergence of the series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$ is infinite.

6. **Show** whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$ converges or diverges.

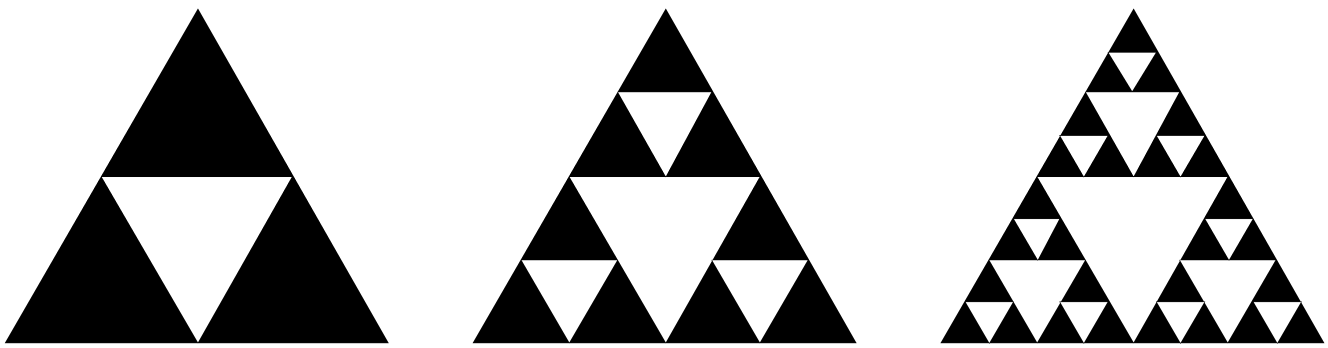
7. Find the **interval of convergence** of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

8. For what values of p does the series $\sum_{n=1}^{\infty} n\left(1+n^2\right)^p$ converge?

9. Chaz is a calculus student at E.S.U. who's having some trouble with series. Chaz says "Dude, this calc stuff is totally destroying me. Like, I figured those series would pretty much be just common sense, but I totally bombed the test. But there's this one where I think I got screwed, because my way totally makes sense. I showed how this one series they asked about, it was like one over n minus one over the whole n minus 1, so it was a thing that diverged minus a thing that diverged, and that meant it had to diverge totally, right? But the grader gave me, like, no credit, and wrote this stuff about how that's bad math and stuff. I think he just hates me because I'm in a frat."

Explain clearly, in terms Chaz can understand, either why his approach is incorrect or why it's valid and how he can convince the grader.

10. Begin with an equilateral triangle with a total area of 1 and successively remove smaller triangles from it in the manner shown below:



(a) If a_n is the total area removed in step n alone, find a_1, a_2, a_3, and a_4.

$a_1 =$

$a_2 =$

$a_3 =$

$a_4 =$

(b) If we continue the process indefinitely, express the total area removed as a series and find the sum of that series.

Extra Credit (5 points possible): Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$.