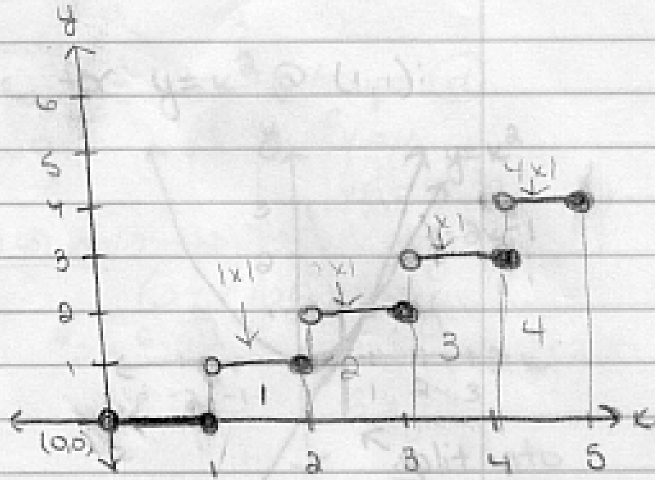
① ⓐ first draw a graph:
try different #s for n &
look for pattern:



$\int_0^2 [\![x]\!]dx = 0+1 = 1$

$\int_0^3 [\![x]\!]dx = 0+1+2 = 3$

$\int_0^5 [\![x]\!]dx = 0+1+2+3+4 = 10$

so....

$$\int_0^n [\![x]\!]\,dx = (n-1) + (n-2) \ldots\ldots + (n-n)$$

ⓑ try #s again (look @ same graph):

$\int_1^2 [\![x]\!]dx = 1$

$\int_2^5 [\![x]\!]dx = 2+3+4 = 9$

$\int_3^6 [\![x]\!]dx = 3+4+5 = 12$

Great

so.....

$$\int_m^n [\![x]\!]dx = m + (m+1) + (m+2) \ldots\ldots (n-1)$$

② ⓐ equation of tangent line for $y=x^2$ @ $(1,1)$:

$y=x^2$

$y'=2x$

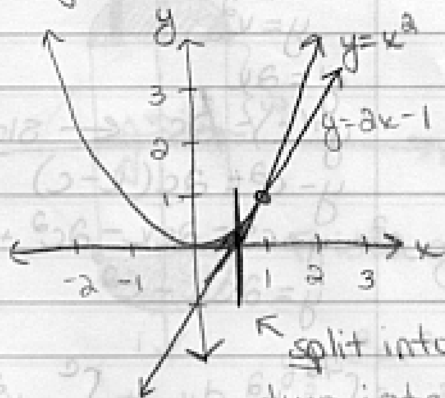
$y'(1)=2(1)$

$y'(1)=2$ ← slope @ $(1,1)$

$y-1=2(x-1)$

$y-1=2x-2$

$\underline{y=2x-1}$



↖ split into two integrals

good

$$\int_0^{1/2} x^2\,dx + \int_{1/2}^{1} x^2-2x+1\,dx$$

$2x-1=0$

$\frac{2x}{2}=\frac{1}{2}$

$x=1/2$

$$\left[\frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3}-x^2+x\right]_{1/2}^{1}$$

$$\left[\frac{(1/2)^3}{3}\right] + \left\{\left[\frac{1^3}{3}-1^2+1\right]-\left[\frac{(1/2)^3}{3}-(1/2)^2+1/2\right]\right\}$$

$$\frac{1/8}{3} + \left\{\left[\frac{1}{3}-1+1\right]-\left[\frac{1/8}{3}-\frac{1}{4}+\frac{1}{2}\right]\right\}$$

$$\frac{1}{24} + \left\{\frac{1}{3}-\left[\frac{1}{24}-\frac{1}{4}+\frac{1}{2}\right]\right\}$$

$$\frac{1}{24} + \left\{\frac{8}{24}-\left[\frac{1}{24}-\frac{6}{24}+\frac{12}{24}\right]\right\}$$

$$\frac{1}{24} + \left\{\frac{8}{24}-\frac{7}{24}\right\}$$

$$\frac{1}{24}+\frac{1}{24}$$

$$\frac{2}{24} = \frac{1}{12}$$

(b) equation of tangent line of $y=x^2$ @ (c,c^2)

$y=x^2$

$y'=2x$

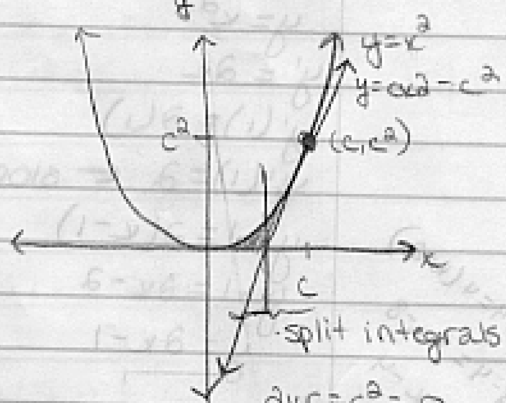
$y'(c)=2c \leftarrow$ slope @ (c,c^2)

$y-c^2=2c(x-c)$

$c^2+y-c^2=2cx-2c^2+c^2$

$y=2cx-c^2$

Nice!



$2xc-c^2=0$

$c(2x-c)=0$

$2x-c=0$

$\frac{2x}{2}=\frac{c}{2}$

$$\int_0^{c/2} x^2\,dx+\int_{c/2}^{c} x^2-2cx+c^2\,dx$$

$$\left[\frac{x^3}{3}\right]_0^{c/2}+\left[\frac{x^3}{3}-cx^2+c^2x\right]_{c/2}^{c}$$

$$\left[\frac{(c/2)^3}{3}\right]+\left\{\left[\frac{c^3}{3}+\cancel{c^3}+\cancel{c^3}\right]-\left[\frac{(c/2)^3}{3}-c(c/2)^2+c^2(c/2)\right]\right\}$$

$$\left(\frac{c^3/8}{3}\right)+\left\{\frac{c^3}{3}-\left(\frac{c^3/8}{3}-c\left(\frac{c^2}{4}\right)+\frac{c^3}{2}\right)\right\}$$

$$\cancel{\frac{c^3}{24}}+\left\{\frac{c^3}{3}+\left(\cancel{\frac{-c^3}{24}}+\frac{c^3}{4}+\frac{-c^3}{2}\right)\right\}$$

$$\frac{c^3}{3}+\frac{c^3}{4}-\frac{c^3}{2}$$

$$\frac{4c^3}{12}+\frac{3c^3}{12}-\frac{6c^3}{12}$$

$$\frac{c^3}{12}$$

(5) $V = 2\pi \int_0^{h/2} \left\{ \left(\sqrt{r^2 - x^2}\right)^2 - \left(\sqrt{r^2 - (h/2)^2}\right)^2 \right\} dx$

$V = 2\pi \int_0^{h/2} \left[(r^2 - x^2) + (r^2 + h^2/4) \right] dx$

$V = 2\pi \int_0^{h/2} -x^2 + \frac{h^2}{4}\, dx$

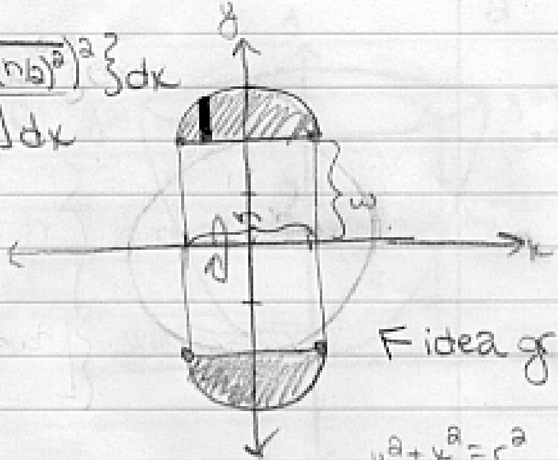
$V = 2\pi \left[\frac{-x^3}{3} + \frac{h^2 x}{4} \right]_0^{h/2}$

$V = 2\pi \left\{ \left[\frac{-(h/2)^3}{3} + \frac{h^2(h/2)}{4} \right] \right\}$

$V = 2\pi \left[\frac{-h^3}{24} + \frac{3h^3}{24} \right\}$

$V = 2\pi \cdot \frac{2h^3}{24}$

$$V = \frac{\pi h^3}{6}$$

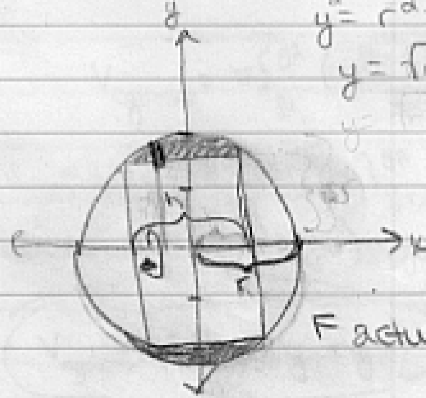


F idea graph

$y^2 + x^2 = r^2$

$y^2 = r^2 - x^2$

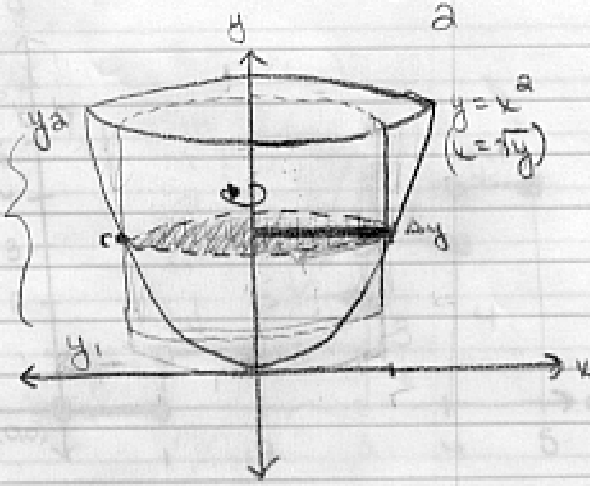
$y = \sqrt{r^2 - x^2}$



F actual graph used

Great

$r = \frac{y_2 + y_1}{2}$



(4) $V_{frustum} = \pi \int_{y_1}^{y_2} (\sqrt{y})^2 \, dy$

$V = \pi \int_{y_1}^{y_2} y \, dy$

$V = \pi \left[\frac{y^2}{2} \right]_{y_1}^{y_2}$

$V = \pi \left[\frac{y_2^2}{2} - \frac{y_1^2}{2} \right]$

Excellent!

$V = \pi \left(\frac{y_2^2 - y_1^2}{2} \right)$

$V = \frac{\pi}{2} (y_2 + y_1) \underbrace{(y_2 - y_1)}_{\text{same as } h}$

$V_{frustum} = \frac{\pi h}{2} (y_2 + y_1)$

$V_{cylinder} = \pi \int_{y_1}^{y_2} \left(\frac{y_1 + y_2}{2} \right)^2 dy$

$V = \pi \frac{y_1 + y_2}{2} [y]_{y_1}^{y_2}$ same as h

$V = \frac{\pi}{2} (y_1 + y_2)(y_2 - y_1)$

$=$

$V_{cylinder} = \frac{\pi h}{2} (y_1 + y_2)$