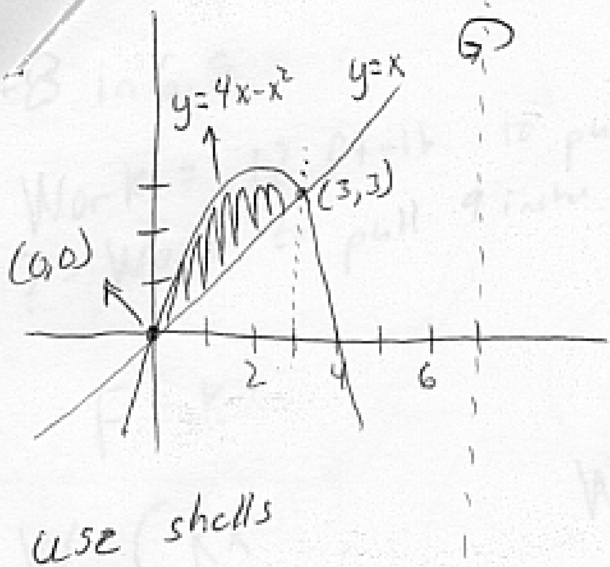
Problem Set 2

about $x = 7$



use shells

$$V_{solid} = 2\pi \int_a^b (hgt)(radius)\, dx$$

5 $$V_{solid} = 2\pi \int_0^3 (4x - x^2 - x)(7-x)\, dx$$

$$V_{solid} = 2\pi \int_0^3 (3x - x^2)(7-x)\, dx$$ Yes

$$V_{solid} = 2\pi \int_0^3 (x^3 - 10x^2 + 21x)\, dx$$

$$V_{solid} = 2\pi \left[\frac{x^4}{4} - \frac{10x^3}{3} + \frac{21x^2}{2} \right]_0^3$$

$$= 2\pi \left[\frac{81}{4} - 90 + \frac{21(9)}{2} \right]$$

$$= 2\pi \left[99/4 \right]$$

$$= \frac{99\pi}{2}$$

$$V = \boxed{49.5\pi \text{ units}^3}$$

①2. Here,

When the spring is being streched 1ft beyond its natural length,

Work done = 12 ft-lb

Now,

$$W = \int_0^1 f(x)\,dx = \int_0^1 Kx\,dx = 12$$

or, $K\left[\frac{x^2}{2}\right]_0^1 = 12$

or, $\frac{K}{2} = 12$ $\therefore$ $\underline{K = 24}$

Here, it is streched 9 inches beyond its natural length.

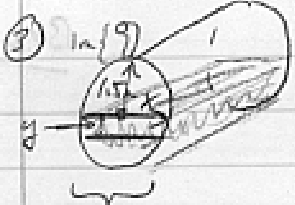
9 inches $= \frac{3}{4}$ ft

So, $\underline{W = \int_0^{3/4} Kx\,dx} = \underline{\int_0^{3/4} 24x\,.dx}$

$= 24\left[\frac{x^2}{2}\right]_0^{3/4} = \frac{27}{4} = \underline{6\,{}^{3}/_{4}}$

Well done

③



$$F_{slice} = 2\sqrt{r^2-y^2}\,(920)(6)\Delta h$$

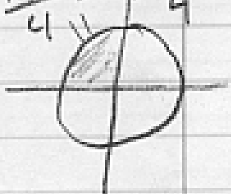
$$W_{slice} = F\Delta x = 2\sqrt{r^2-y^2}\,(920)(6)(2.5-y)\Delta h$$

$r^2 = x^2+y^2 \Rightarrow 2\sqrt{r^2-y^2}$

$$W_{total} = \left(\int_{-1.5}^{0} 2\sqrt{r^2-y^2}\,(920)(6)(2.5-y)\,dy\right)9.8$$

$$(9.8)(2)(920)(6)\left[\int_{-1.5}^{0} 2.5\sqrt{r^2-y^2} - \int_{-1.5}^{0} y\sqrt{r^2-y^2}\,dy\right]$$

$\frac{\pi r^2}{4} = \frac{\pi(1.5)^2}{4}$



$$= (9.8)(2)(920)(6)\left[\left(\frac{\pi(1.5)^2}{4}\right)(2.5) - \int_{-1.5}^{0} y\sqrt{u}\,\frac{du}{-2y}\right]$$

let $u = (1.5)^2 - y^2$

$du = -2y\,dy$

$dy = \frac{du}{-2y}$

$$= (9.8)(2)(920)(6)\left[\frac{\pi(1.5)^2}{4}(2.5) + \frac{1}{2}\int_{-1.5}^{0} u^{1/2}\,du\right]$$

$$= (9.8)(2)(920)(6)\left[\frac{\pi(1.5)^2}{4}(2.5) + \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)_{-1.5}^{0}\right]$$

$$= 152145\pi + 54096\left[\frac{2}{3}(2.25-y^2)^{3/2}\right]_{-1.5}^{0}$$

$$= 152145\pi + 54096\left[\frac{2}{3}(2.25-0)^{3/2} - \frac{2}{3}\cancel{(2.25-2.25)}^{3/2}\right]$$

$$= 152145\pi + 54096\left[\frac{2}{3}(2.25)^{3/2}\right]$$

$$\boxed{W = (152145\pi + 121716)\ \text{J}}$$

(14) $\dfrac{\int_0^b 2+6x-3x^2}{b-0} = 3$ Great

$\dfrac{\left[2x+3x^2-x^3\right]_0^b}{b} = 3$

$\dfrac{2b+3b^2-b^3}{b} = 3$

$\dfrac{2b}{b} + \dfrac{3b^2}{b} - \dfrac{b^3}{b} = 3$

$2 + 3b - b^2 = 3$

$-b^2+3b+2=3$

$-b^2+3b-1=0$

$b^2-3b+1=0$

Use quadratic formula:

$b = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

where $a=1$, $b=-3$, $c=1$

$b = \dfrac{3 \pm \sqrt{9-(4\cdot 1\cdot 1)}}{2(1)}$

$b = \dfrac{3 \pm \sqrt{5}}{2}$

$b = \dfrac{3+\sqrt{5}}{2}, \dfrac{3-\sqrt{5}}{2}$