Exam 2   Calc 2   2/20/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Set up** an integral for the area of the first-quadrant portion of a circle centered at the origin with a radius of 5.

2. Jon is considering a big investment in black-market software distribution. He thinks the investment will pay $5000 per year initially, but drop off linearly to $0 after 2 years. **Set up an integral** that gives a fair price to pay for this investment today (as an alternative to investing the money at 6% continuously compounded interest)?

3. A spring with natural length 40cm requires 5N of force to hold it stretched to a length of 50cm. **How much work** is done in stretching the spring from natural length to a length of 60cm?

4. Suppose that the probability density function for the failure of the airbag system in Jon's car after t months is given by $p(t) = ce^{-ct}$ for values of t greater than zero (and by zero otherwise).

(a) If the probability of failure within the first month is 40%, **find the value** of c to two decimal places.

(b) **Write an integral** which expresses the probability of the airbag system failing within the first 6 months.

5. A thin metal rod 5 inches long has density given by $d(x) = \sqrt{4+x}$ at a point x inches from its left end. **Set up an integral** and use it to find the mass of the rod.

6. Bunny is a calculus student at Factory State University, and she's having some trouble. Bunny says "This is a disaster! I'm so totally gonna fail calculus, and Daddy's gonna take away my clothes allowance! Every time I think I've got it, it turns out I'm totally wrong. There was this problem on the test about finding the volume of the top six inches of a sphere with radius 12 inches, right? So I said easy, I know the formula for the volume of the whole sphere, and I'll just divide that by 4 to get the top quarter of it, right? But they gave me no credit at all for it. How am I gonna buy those new shoes?"

Explain clearly to Bunny why what she did does or doesn't work, and how she should think about the problem.

7. **Set up an integral** for the arc length of the curve $y = 1/x$ from the point $(2, \frac{1}{2})$ to the point $(5, \frac{1}{5})$.

8. Jon has a piece of cardboard cut in the shape of the region between $x = 9 - y^2$ and the y axis. Set up integrals and use them to **find** $\overline{x}$, the x coordinate of the center of mass of the cardboard.

9. A spherical tank with a radius of 4 feet is buried so that its top is 6 feet underground. If the tank is full of water (with a density of 62.4 pounds per cubic foot), **write an integral** for the amount of work required to pump half the water up to the surface.

10. **Evaluate** $\displaystyle\int_0^1 \ln x \, dx$.

Extra Credit (5 points possible): The surface obtained by rotating the region between y = 1/x and the x axis to the right of x = 1 around the x axis is called *Gabriel's Horn*.

(a) Set up an integral for the volume of Gabriel's Horn and evaluate it to find its volume.

(b) The integral $\displaystyle 2p\int_a^b f(x)\sqrt{1+\left[f'(x)\right]^2}\, dx$ gives the surface area of the solid obtained by

revolving the region between f(x) and the x axis between x = a and x = b around the x axis. Find the surface area of Gabriel's Horn.