

**Exam 2b    Calc 2    2/20/2004**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. **Write an integral** for the area of the region between the curve  $y = 9 - x^2$  and the  $x$  axis.

2. If an investment pays interest at a continuous rate of 6%, and you deposit money at a constant rate of \$5000 per year, **write an integral** that gives the amount of money in the investment when you move to the Carribean to retire 20 years from now.

3. A spring with natural length 9 inches requires 10 foot-pounds of work to stretch it from a length of 9 inches to 12 inches. **How much work** is done in stretching the spring from 12 inches to a length of 18 inches?

4. **Set up integrals** for  $\bar{x}$ , the  $x$  coordinate of the center of mass of the ellipse with equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

5. Suppose the probability density function for the length of time it takes a student to finish a math test is

given by 
$$p(x) = \begin{cases} bx^3 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} .$$

(a) Find the exact value of  $b$ .

(b) Find the mean time it takes a student to complete the test.

6. Set up an integral for the volume of a cone with height 5cm and base diameter 4cm, and use it to calculate the volume of the cone to the nearest cubic centimeter.

7. **Set up an integral** for the length of the curve  $y = \ln x$  from the point  $(1,0)$  to the point  $(e,1)$ .

8. Bunny is a calculus student at Factory State University, and she's having some trouble. Bunny says "This is a disaster! I'm so totally gonna fail calculus, and Daddy's gonna take away my clothes allowance! I just failed our test, and unless I can get this extra credit I'm doomed. We're supposed to write an explanation about this center of mass stuff, like if somebody said that a rod that was 4 feet long had its center of mass right at the middle, does that mean it had to have the same weight in the left two feet as in the right two feet. I think it must be true, right, because that's the only way it would balance is if the part on the right had the same weight as the part on the left, isn't it?"

Explain clearly to Bunny whether it's true or false that a rod which balances at its midpoint must have the same amount of weight on its left and right, and why.

9. A spherical tank with a radius of 4 feet is buried so that its top is 6 feet underground. If the tank is half full of water (with a density of 62.4 pounds per cubic foot), **write an integral** for the amount of work required to pump the water up to the surface.

10. Evaluate  $\int_1^{\infty} \frac{\ln x}{x^2} dx$ .

Extra Credit (5 points possible): The integral  $2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2} dx$  gives the surface area of the solid obtained by revolving the region between  $f(x)$  and the  $x$  axis between  $x = a$  and  $x = b$  around the  $x$  axis. Use it to find the surface area of a sphere with radius  $R$  [Hint: You'll want to view the sphere as the result of revolving a circle around the axis, and as usual it might help to warm up with a radius of just 1].