Practice Exam 2   Calc 2   2/17/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.
Warning: This is only a practice exam. Knowing how to do the problems on this exam is good, but if
that's **all** you know, you're not likely to do well at all on the actual exam. Use this as a warm-up, a
trial run, or just to get used to the form questions might take, but don't think of it as a complete study
guide.

1. Suppose that cars pass Coe's campus on First Avenue with a density of 200 cars per hour at 4am
one morning, and this increases linearly to 1000 cars per hour at 6am. How many cars pass during the
period from 4am to 6am?

2. If the work required to stretch a spring 1 foot beyond its natural length is 15 ft-lb, how much
work is needed to stretch it 6 inches beyond its natural length?

3. You have a big opportunity to buy an ostrich ranch. If you expect a revenue of $50,000 per year to start immediately and increase steadily up to a level of $100,000 per year after 10 years, write an integral which gives the present value of the revenue from the ranch over the next 10 years (assume 6% continuously compounded interest).

4. Set up an integral for the length of the curve $y = \sin x$ between the point $(0,0)$ and the point $(\pi,0)$.

5. If the probability density function for the length of time it takes a baby lynx to stray more than 1 foot from its mother after birth is given by $p(t) = 0.32e^{-0.32t}$, where t is measured in days, what is the median time it takes a baby lynx to stray more than 1 foot from its mother?

6. The graph of $x^2 - y^2 = 1$ is a hyperbola. Set up an integral for the area in the first quadrant bounded by the hyperbola and the line $y = 1$ and use it to find the area.

7. Brandi is a calculus student at E.S.U. who's having some trouble with improper integrals. Brandi says "So there was this problem on our test, and it was to say if one over $x \ln x$ converged or not. I drew this total blank on how to find the antiderivative, but I thought about it and decided I didn't really have to. See, you know that if you integrate $1/x$ it's infinity, but if you integrate $1/x^2$ or anything else where the denominator is more than just x it converges. So I just said all that and said it must converge. But the grader gave me no points at all, and just said that didn't work, but not why, and then he told me to go away because he hates dealing with students."

Explain to Brandi either why she's right, or what's wrong with her reasoning.

8. Jon has a bowl-shaped fountain on his desk which is shaped like a frustum of a sphere with a radius of 9 inches, cut off 3 inches up from the bottom. Set up an integral and use it to find the volume of water contained in this fountain.

9. **Find** the x coordinate of the centroid of the trapezoidal region with vertices at $(0,0)$, $(a,0)$, $(0,b)$, and (a,c).

10. A well with circular horizontal cross-sections is 40 feet deep and narrows from 10 feet in diameter at the surface to 6 feet in diameter at its bottom. If the well has 8 feet of water at the bottom, how much water will 1000 foot-pounds of work pump to the top of the well?

Extra Credit (5 points possible): The greatest integer function $g(x) = [\![x]\!]$ returns the greatest integer which is less than or equal to the input, so for instance $[\![3]\!] = 3$ and $[\![2.4]\!] = 2$. If the region under the graph of $y = [\![x]\!]$ but above the x axis, between $x = 0$ and $x = m$ (for some integer m), is rotated around the x axis, what can you say about the volume of the resulting solid?