Exam 3b Calc 2 3/26/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find the sum of the series $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\ldots$

2. Write the series $(x-2)+\frac{(x-2)^2}{3}+\frac{(x-2)^3}{5}+\frac{(x-2)^4}{7}+\ldots$ in sigma notation.

3. Write a fifth degree Taylor polynomial for $\dfrac{\cos x - 1}{x^2}$ centered at zero.

4. The first three derivatives of the function $f(x) = \arccos x$ are listed below. Use them to find the 3rd degree Taylor polynomial for $\arccos x$ centered at $x = 0$.

$$f'(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$f''(x) = \frac{-x}{\left(1-x^2\right)^{3/2}}$$

$$f'''(x) = \frac{-1}{\left(1-x^2\right)^{3/2}} - \frac{3x^2}{\left(1-x^2\right)^{5/2}}$$

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2}$ converges or diverges.

6. Is x = 1 included in the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$?

7. What is the radius of convergence of the series $\sum_{n=0}^{\infty}\left(\frac{x}{4}\right)^n$?

8. Biff is having trouble with calculus again. He says "Dude, these series are kicking my ass. There was this one thing, where, like, we were supposed to say something about using a series to approximate something, right? And like, what would happen if you were putting in something outside the interval of convergence, right? So I said that then you'd have to use really a lot of terms, like a really high degree, to make it accurate, you know? But the guy who sits next to me said he didn't think that was right."

Explain to Biff whether using a higher degree polynomial approximation will help outside the radius of convergence, and why.

9. Does the series $1-\frac{2}{3}+\frac{3}{5}-\frac{4}{7}+\frac{5}{9}-\ldots$ converge? Justify your answer.

10. Find the fifth degree Taylor polynomial for $\sin(\tan x)$. [Hint: The derivatives involved are ridiculous – find a better way.]

Extra Credit (5 points possible): Evaluate $1+\frac{1}{2}-\frac{2}{3}+\frac{1}{4}+\frac{1}{5}-\frac{2}{6}+\frac{1}{7}+\frac{1}{8}-\frac{2}{9}+\frac{1}{10}+\ldots$