

Exam 4b Calc 2 4/23/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Determine whether $y = 3e^{-2t}$ is a solution to the differential equation $\frac{d^2 y}{dt^2} = 9y$.

2. Find a general solution to the differential equation $\frac{dy}{dx} + xy^3 = 0$.

3. Which of the differential equations could have the slope field pictured at right?

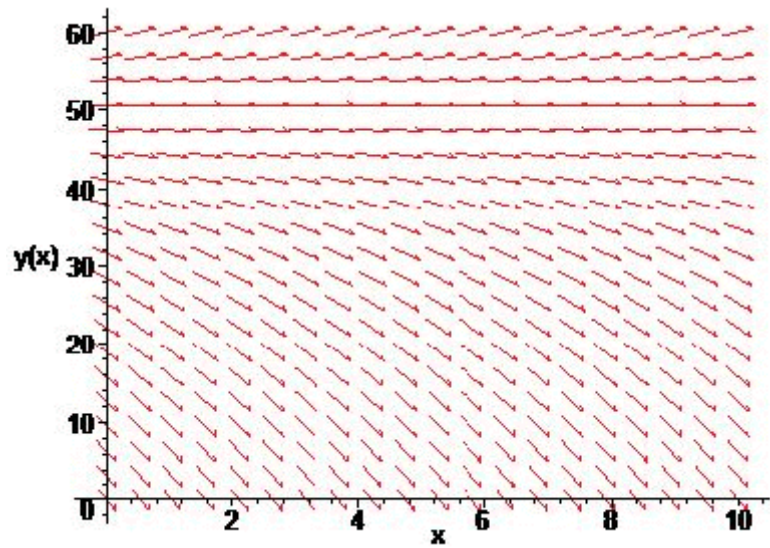
a) $\frac{dp}{dt} = 0.2p$

b) $\frac{dp}{dt} = -0.2p$

c) $\frac{dp}{dt} = 0.2p + 20$

d) $\frac{dp}{dt} = 0.2p - 10$

e) $\frac{dp}{dt} = 0.2p(50 - p)$



4. Find a general solution for the differential equation $y'' + 5y' + 6y = 0$.

5. Find all equilibrium points of the system

$$\frac{dx}{dt} = x - x^2 - \frac{xy}{3}$$

$$\frac{dy}{dt} = y - y^2 - \frac{xy}{2}$$

6. Suppose a 70° kumquat is placed in an oven that begins at 70° but which heats linearly to 350° over the next 10 minutes. Write a differential equation for the temperature of the kumquat after t minutes (use 0.0673 for k) and use Euler's method with $\Delta t = 5$ minutes to estimate the temperature (correct to 2 decimal places) of the kumquat at the end of 10 minutes.

7. Bunny is having trouble with differential equations. She says “Ohmygod! It’s so totally unfair! I mean, I can do math okay when they give the directions right, you know? But now they totally just don’t tell us what to do, and I’m lost. They said to pick a suitable kind of differential equation for how many people have cable modems in their houses and use it, but that’s so wrong, because how am I supposed to know? I mean, when they say something’s proportional to something I can do that, but how am I supposed to know what’s proportional to cable modems?”

Suggest to Bunny what sort of model might be appropriate for this situation, and why.

8. Lake Erie has a volume of $460,000 \text{ km}^3$ and an outflow rate of 175 km^3 per year. Suppose that 20 kg of a certain pollutant is present in the lake at time 0 . How long (to the nearest year) will it be until only 5 kg remain?

9. Suppose that the population of carp in a certain river grows logistically with a carrying capacity of 8000 fish, and that when there are 6000 fish in the river the population grows at a rate of 480 fish per year. How many fish can safely be harvested from the river without causing extinction?

10. The differential equation $y'' + 4y = 3 \cos 5t$ isn't quite the sort where we can use our standard characteristic polynomial strategy, but it's close. Suppose that there's a solution of the form $y = a \cos 5t$ for some value of the constant a , and see if you can find a workable value for a .

Extra Credit (5 points possible): Try to find a solution $y(t)$ of the differential equation $y'' - ty' - 2y = 0$.