Quiz 1a Calculus 2 1/21/04

Each problem is worth 5 points. For full credit provide proper justification for your answer.

1. Find the value of $\int_0^{\infty} \frac{x}{e^x}\,dx$ or show that it diverges.

$\int x \cdot e^{-x}\,dx$ let $u = x$ $v = -e^{-x}$

$u' = 1$ $v' = e^{-x}$

$= -xe^{-x} - \int -e^{-x}\,dx$

$\left[-xe^{-x} - e^{-x}\right]_0^b$

$= \lim_{b\to\infty}\left[\left(\frac{-b}{e^b} - \frac{1}{e^b}\right) - \left(\frac{-0}{e^0} - \frac{1}{e^0}\right)\right]$

$= \lim_{b\to\infty} \frac{-b}{e^b} - 0 + 0 + 1$

$\overset{L'H}{=} \lim_{b\to\infty} \frac{-1}{e^b} + 1 = 0 + 1 = 1$

2. Find the area of the region bounded between $y = 2x$ and $y = x^2 - 3$.

$\int_{-1}^{3} \left[2x - (x^2 - 3)\right]dx$

$2x = x^2 - 3$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$= \left[x^2 - \frac{1}{3}x^3 + 3x\right]_{-1}^{3}$

$= (9 - 9 + 9) - (1 + \frac{1}{3} - 3)$

$= 9 + 5/3$

$= \frac{27}{3} + \frac{5}{3}$

$= \frac{32}{3}$