

Each problem is worth 5 points. For full credit provide proper justification for your answer.

1. Find a **general** solution to the differential equation $y'' + y' - 6y = 0$.

assume $y = e^{s \cdot t}$

$$y' = s e^{s t}$$

$$y'' = s^2 e^{s t}$$

$$s^2 e^{s t} + s e^{s t} - 6(e^{s t}) = 0$$

$$e^{s t} (s^2 + s - 6) = 0$$

$$e^{s t} (s+3)(s-2) = 0$$

$$s = -3 \quad s = 2 \quad \text{so}$$

$$y = a e^{-3t} + b e^{2t}$$

Excellent

2. If you know that the differential equation $y'' + 3y' + 2y = 0$ has the general solution $y = a e^{-t} + b e^{-2t}$, find a **particular** solution that satisfies the conditions $y(0) = 0$ and $y'(0) = 1$.

We have,

$$y = a e^{-t} + b e^{-2t}$$

But, $y(0) = 0$

$$\text{So, } 0 = a e^{-0} + b e^{-2 \cdot 0}$$

$$\text{or, } \underline{a + b = 0} \quad \text{or, } a = -b$$

Now,

$$y' = -a e^{-t} - 2b e^{-2t}$$

Also, $y'(0) = 1$

$$\text{So, } \underline{1 = -a e^{-0} - 2b \cdot e^{-2 \cdot 0}}$$

$$\text{or, } \underline{1 = -a - 2b}$$

But, $a = -b$

$$\text{So, } \underline{1 = b - 2b}$$

$$\text{or, } \underline{1 = -b}$$

$$\therefore \underline{b = -1}$$

$$\therefore \underline{a = 1}$$

Nice Job!

$$y = e^{-t} - e^{-2t}$$