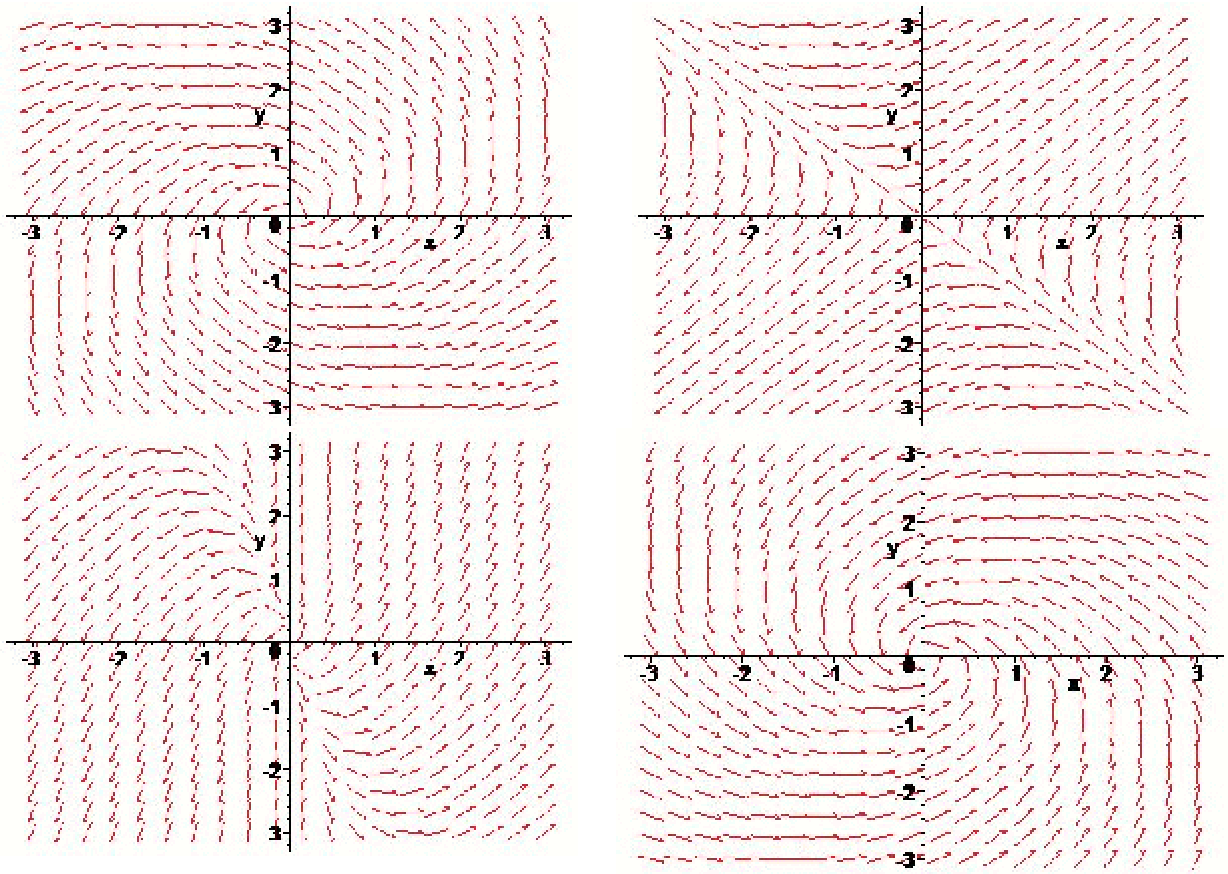
Exam 3 Differential Equations 4/9/04

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Give an example of eigenvalues that would produce a saddle at the origin.

2. If you know that a planar system has eigenvalues $\lambda_1 = 4$ and $\lambda_2 = 0$, with corresponding eigenvectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, write a general solution for the system.

3. Which of the phase planes shown below could correspond to a planar system with eigenvalues $\lambda_1 = 2 + i$ and $\lambda_2 = 2 - i$?



4. Find a **general** solution to the system

$$\begin{aligned} \frac{dx}{dt} &= -2x + y \\ \frac{dy}{dt} &= \phantom{-2x + {}} 3y \end{aligned}.$$

5. The planar system $\begin{aligned}\frac{dx}{dt} &= 2x - 3y \\ \frac{dy}{dt} &= 3x + 2y\end{aligned}$ has eigenvalues $\lambda_1 = 2 + 3i$ and $\lambda_2 = 2 - 3i$, and corresponding eigenvectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$. Write a general solution to this system in a form not involving imaginary exponents.

6. For which values of the constant b will the system of differential equations $\begin{aligned}\frac{dx}{dt} &= 2x + by \\ \frac{dy}{dt} &= 4x + 6y\end{aligned}$ have solutions involving sine and cosine?

7. Find a particular solution to the planar system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -2 & -1 \\ 1 & -4 \end{pmatrix}\mathbf{Y}$ which satisfies the initial condition $\mathbf{Y}_0 = (1,0)$.

8. Suppose that the planar system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -0.2 & 0.1 \\ 0.0 & -0.1 \end{pmatrix}\mathbf{Y}$ represents the populations of two species of fish, creatively labeled as “species X” and “species Y”, where x(t) actually gives the number of fish of species X in a pond above or below the natural equilibrium population and y(t) gives the number of fish of species Y present in the pond.

The eigenvalues of the system are $\lambda_1 = -0.2$ and $\lambda_2 = -0.1$, with associated eigenvectors $\mathbf{V}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{V}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(a) Does the presence of fish of species Y benefit or hurt the population of species X?

(b) If all the fish of species Y are exterminated, what will happen to species X?

(c) If a small number of fish of species Y are introduced, with species X beginning at equilibrium, what will happen?

9. Convert the second-order differential equation $ay'' + by' + cy = 0$ to a system of first-order equations and find the equation in λ whose roots are the eigenvalues of the system.

10. Jon is trying to make up a problem for a differential equations test. He wants a planar system with eigenvalues 2 and 3, with respective eigenvectors $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. He knows that you can make a matrix with desired eigenvalues by putting the eigenvalues you want as entries on the main diagonal of the coefficient matrix with a zero in one of the other spots, but what coefficient matrix does he need in order to have his desired eigenvectors?