Exam 2a   Calc 2   2/25/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Set up an integral for the area of the portion of the circle $x^2 + y^2 = 9$ which lies to the right of the line $x = 1$.

2. Set up an integral for the volume of the solid obtained by rotating the region bounded by $y = 0$, $y = x^2$, and $x = 5$ around the x axis.

3. A force of 10 pounds is required to hold a spring stretched 6 inches beyond its natural length. How much work, to the nearest tenth of a foot-pound, is done in stretching it from 4 inches to 12 inches beyond its natural length?

4. Set up an integral for the arc length of the portion of the curve $y = x^3$ between the points (-1,-1) and (2,8).

5. Set up an integral or integrals for the x coordinate of the center of mass of the right-hand portion of the region bounded by $y = x^3$ and $y = 4x$.

6. If $p(x) = \begin{cases} 0.3e^{-0.3x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$ is a probability density function giving the probability of a baby rhinoceros taking its first steps x days after birth, compute the median number of days after birth for a rhinoceros to take its first steps.

7. Bunny is a calculus student at Anonymous State University, and she's having some trouble with probability. Bunny says "Ohmygod, calculus has gotten *so* hard. There's this probability stuff, you know? And like, it's so confusing? My notes are a total mess, and I don't know what they mean at all. It's something like, maybe, like, if you integrate it from zero to one, then it's supposed to be infinite, I think. But I don't know what that means!"

Explain clearly to Bunny a couple of key things to know about probability density functions, including at least one thing involving the "zero to one" interval she mentions, and at least one thing involving infinity.

8. Congratulations, you've been handed the lucky exam! This exam is worth $100, payable to the original recipient of this exam on this date 100 years from now (this offer does not extend to any individual other than the original recipient, and is not transferrable to any heirs or assignees). What constant income stream (to the nearest hundredth of a cent), payed out over the next 100 years and assuming continuous interest at 8%, would be of equivalent value?

9. Jon is planning to go into business selling new underground gasoline storage tanks to gas stations. The tanks will be shaped like the picture shown below, where the front and back faces are shaped like the parabola $y = x^2$ between (-2,4) and (2,4), and from front to back the tank measures 10 feet. Jon intends to market these tanks based on the claim that since more of the tank is near the surface than in a standard storage tank shaped like a cylinder lying on its side, the amount of work the pumps need to do will be less with these tanks than with conventional tanks. Set up an integral for the amount of work required to pump all the gasoline in one of Jon's tanks to a point 6 feet above the top of the tank (gasoline weighs 42 lb/ft^3).
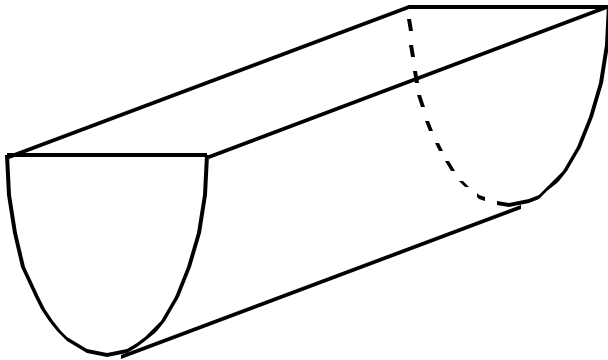
10. Suppose that $f(x)$ is a function (with $f(x) \geq 0$ everywhere) whose length from $x = a$ to $x = b$ is given by L, and that the surface area of the solid obtained by rotating the region below $f(x)$ around the x axis is given by S. Let $g(x)$ be another function, with $g(x) = f(x) + 2$. Express the surface area of $g(x)$ in terms of the values L and S.

Extra Credit (5 points possible):

The solid obtained by revolving the curve with equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ around the x axis is called a hyperboloid of two sheets. Compute the volume of the portion of a hyperboloid of two sheets which lies between $x = a$ and $x = c$.