Exam 3a Calc 2 4/1/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Write a 5th degree Taylor polynomial for the function $f(x) = \sin x$ centered at $x = 0$.

2. Find the sum of the series $\frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} + \ldots$

3. Show that $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

4. Use a 4th degree Taylor polynomial to find an approximation of $\cos 0.2$ to 8 decimal places.

5. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{(2n)!}$.

6. Biff is a calculus student at Anonymous State University, and he's having some trouble with series. Biff says "Man, this convergence stuff is kicking my ass. We just had our test over it and I guess everybody failed, 'cause the professor said we get to take it over again next week to make our grades better. So one of the questions was about whether approximations from Taylor polynomials are always accurate if you use a high enough degree polynomial. It was multiple choice, and I picked the answer that said 'yeah, as long as you don't have an arithmetic mistake', 'cause that's where I screw up a lot. But the machine scored it wrong, which I think is crap, because obviously it's true for me, right?"

Explain clearly to Biff why his answer is actually correct or not, and what sort of answer his professor likely counts as correct.

7. Students occasionally insist that $\sin^2 x$ is the same as $\sin(x^2)$. Show that the Taylor polynomials for these two functions are different.

8. Suppose that $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive terms. Define a new series $\sum_{n=1}^{\infty} b_n$ by letting $b_n = \begin{cases} a_n & \text{if n is odd} \\ 0 & \text{if n is even} \end{cases}$. Does $\sum_{n=1}^{\infty} b_n$ converge?

9. Determine whether $x = {}^{-1}\!/_{3}$ is in the interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(3x)^n}{n^2+1}$.

10. Use the following pieces of information to find a Taylor polynomial of degree 6 for the function cosh x:

- $(\cosh x)' = \sinh x$
- The 5th degree Taylor polynomial for sinh x is $x + \frac{x^3}{6} + \frac{x^5}{120}$ (yes, all plus signs).
- cosh 0 = 1.

Extra Credit (5 points possible):

If f(x) has Taylor series $\sum_{n=0}^{\infty} a_n x^n$ with radius of convergence r, what is the Taylor series for f′(x) and what can you say about its radius of convergence?