Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Circle all of the candidates below which **are** differential equations:

a) $\dfrac{dp}{dt} = kp(500 - p)$

b) $5x = 25$

c) $y = Ae^{kt}$

d) $y'' + 7y' + 12y = 0$

e) $v' = -32.2$

f) $k\dfrac{dy}{dx} + 7y - \ln|500 - t| + C$

2. Determine whether $y = 2 + 3e^{2x + 4}$ is a solution to the differential equation $\dfrac{dy}{dx} = 2y - 4$.

3. Give an example of a differential equation for a population undergoing exponential growth.

4. Find a general solution to the differential equation $y'' - 4y' - 5y = 0$.

5. Find a general solution to the differential equation $\dfrac{dQ}{dt} - \dfrac{Q}{k} = 0$, where k is a constant.

6. Suppose that the population of fish (measured in thousands) in a large lake is governed by

$$\frac{dp}{dt} = 0.004\,p(30 - p) - 1$$

and $p(0) = 25$. Use Euler's method with $\Delta t = 10$ to approximate the population of fish in the lake 30 years from now to the nearest hundred.

7. Bunny is a calculus student at Anonymous State University, and she's having some trouble with differential equations. Bunny says "OHmygod, these are so confusing. We had this problem on our problem set where, like, we did the oily method, and then it asked about if there was an equilibrium, right? So we did the oily part, and it was totally obvious that it was going up slower and slower, right? So I said there wasn't an equilibrium, because it would never actually get there. The grader gave me, like, almost no points for that second part, which is totally wrong, because he said the first part was totally right. So if it doesn't get there, then it's not an equilibrium, right?

Explain clearly to Bunny whether her statements about equilibrium are correct, or if some refinement is in order.

8. In some chemical reactions, the rate at which the amount of a substance changes with time is proportional to the amount present. For example, this is the case as δ-glucono-lactone acid changes into gluconic acid.[1]

a) Write a differential equation satisfied by y, the quantity of δ-glucono-lactone present at time t.

b) If 100 grams of δ-glucono-lactone is reduced to 54.9 grams in one hour, how many grams (to the nearest tenth of a gram) will remain after 3 hours?

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[1]Borrowed from Hughes-Hallett et al, 3rd ed., p. 512.

9. Small raindrops' velocities can be modeled with the differential equation

$$\frac{dv}{dt} = -32.2 + \frac{4.60 \times 10^{-4}}{d^2} v^2$$

where d is the diameter of the raindrop in feet.

a) Find the equilibrium velocity for a raindrop with diameter $d = 1.00 \times 10^{-2}$ feet, expressing your answer in scientific notation correct to 3 significant digits.

b) Find a general solution to this differential equation.

10. We used characteristic polynomials to solve differential equations of the form

$$ay'' + by' + cy = 0$$

but a variation of that approach can be used to solve equations like

$$y'' - 4y' - 5y = \sin x.$$

Find a solution to this equation by guessing that probably there's a solution of the form

$$y = A\sin x + B\cos x$$

for some values of the coefficients A and B, and proceeding to find suitable values for A and B.

Extra Credit (5 points possible):
Suppose that the differential equation in problem 9 is replaced with

$$\frac{dv}{dt} = -32.2 - \frac{4.60 \times 10^{-4}}{d^2}v^3.$$

Find a general solution to this differential equation.