

Exam 4b Calc 2 4/22/2005

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the definition of a differential equation.

2. Determine whether $y = 2 + 3e^{2x-4}$ is a solution to the differential equation $\frac{dy}{dx} = 2y - 4$.

3. Give an example of a differential equation for a population undergoing logistic growth with a carrying capacity of 8000.

4. Find a general solution to the differential equation $y'' + 4y' - 5y = 0$.

5. Find a general solution to the differential equation $\frac{dP}{dt} - aP = b$, where a and b are constants.

6. Suppose a college has an endowment of \$100 million and earns interest at a continuous rate of 6%, but spends money from that endowment at a rate of \$5 million in the coming year, and then \$1 million more than that for each additional year that passes. This situation can be represented by the differential equation $\frac{dp}{dt} = 0.06p - (5 + t)$. Use Euler's method with $\Delta t = 2$ to approximate, to the nearest tenth of a million dollars, the balance in that school's endowment fund after 6 years have gone by.

7. Bunny is a calculus student at Anonymous State University, and she's having some trouble with differential equations. Bunny says "OHmygod, these are so confusing. We had this problem on our problem set where, like, we did the oily method, and then it asked about if there was an equilibrium, right? So we did the oily part, and it was totally obvious that it was going up slower and slower, right? So I said there wasn't an equilibrium, because it would never actually get there. The grader gave me, like, almost no points for that second part, which is totally wrong, because he said the first part was totally right. So if it doesn't get there, then it's not an equilibrium, right?"

Explain clearly to Bunny whether her statements about equilibrium are correct, or if some refinement is in order.

8. We used characteristic polynomials to solve differential equations of the form

$$ay'' + by' + cy = 0$$

but a variation of that approach can be used to solve equations like

$$y'' + 4y' - 5y = e^{3x}.$$

Find a solution to this equation by guessing that probably there's a solution of the form

$$y = Ae^{3x}$$

for some value of the coefficient A , and proceeding to find an appropriate value for A .

9. Water leaks out of a barrel at a rate proportional to the square root of the depth of the water at that time. If the water level starts out at 24 inches and drops to 20 inches in 1 hour, how long (to the nearest minute) will it take for half of the water to leak out of the barrel?

10. Jon plans to spend his summer building elaborate fountain arrangements involving yard gnomes peeing into rain barrels. If the yard gnome trickles water into the barrel at a rate of 3 inches per hour,

- a) What is the new differential equation, and what is its equilibrium depth (to the nearest tenth of an inch)?
- b) Find a general solution to the new differential equation.

Extra Credit (5 points possible):

Try to find a polynomial that serves as a solution to the differential equation $y' - 1 = 2y$.