

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\int \frac{(\ln z)^3}{z} dz$.

Substitution

let $u = \ln z$

$$\int \frac{u^3}{z} \cdot du \cdot z =$$

$$\frac{du}{dz} = \frac{1}{z}$$

$$= \int u^3 \cdot du = \frac{u^4}{4} + C =$$

$$du = \frac{dz}{z}$$

$$\underline{dz = du \cdot z}$$

$$\boxed{= \frac{(\ln z)^4}{4} + C}$$

Nice!

2. Evaluate $\int_1^5 t \ln t dt$.

Int. by Parts!

$$u = \ln t$$

$$v = \frac{t^2}{2}$$

$$u' = \frac{1}{t}$$

$$v' = t$$

$$= \frac{t^2}{2} \cdot \ln t - \int \frac{t^2}{2} \cdot \frac{1}{t} dt$$

$$= \left[\frac{t^2}{2} \ln t - \frac{1}{2} \cdot \frac{t^2}{2} \right]_1^5$$

$$= \left(\frac{25}{2} \ln 5 - \frac{25}{4} \right) - \left(\frac{1}{2} \ln 1 - \frac{1}{4} \right)$$

$$= \frac{25}{2} \ln 5 - \frac{24}{4}$$

$$= \frac{25}{2} \ln 5 - 6$$

$$\text{Check: } \left(\frac{t^2}{2} \ln t - \frac{t^2}{4} \right)'$$

$$= \frac{2t}{2} \cdot \ln t + \frac{t^2}{2} \cdot \frac{1}{t} - \frac{2t}{4}$$

$$= t \ln t \quad \checkmark$$