Quiz 4b **Calculus 2** **3/25/2005**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Find the Taylor polynomial of degree 3 for the function f(x) = $\sqrt[3]{1+x}$ near 0.

$f(x) = (1+x)^{1/3}$ | $f(0) = 1$

$f'(x) = \frac{1}{3}(1+x)^{-2/3}$ | $f'(0) = \frac{1}{3}$

$f''(x) = -\frac{2}{9}(1+x)^{-5/3}$ | $f''(0) = -2/9$

$f'''(x) = \frac{10}{27}(1+x)^{-8/3}$ | $f'''(0) = \frac{10}{27}$

$$f(x) = f(0) + \frac{f'(0)}{1}x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$$\frac{-\frac{2}{9}}{2!} = -\frac{1}{9} \qquad \frac{\frac{10}{27}}{3!} = \frac{5}{81}$$

Great

$$\sqrt[3]{1+x} \approx 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3$$

2. Find the first 4 terms of the Taylor series for g(x) = sin x about the point $x = \pi/4$.

$g(x) = \sin x$ | $g(\pi/4) = \frac{\sqrt{2}}{2}$

$g'(x) = \cos x$ | $g'(\pi/4) = \frac{\sqrt{2}}{2}$

$g''(x) = -\sin x$ | $g''(\pi/4) = -\frac{\sqrt{2}}{2}$

$g'''(x) = -\cos x$ | $g'''(\pi/4) = -\frac{\sqrt{2}}{2}$

Great

$$\sin x \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) + \left(-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)\left(x - \frac{\pi}{4}\right)^2 + \left(-\frac{\sqrt{2}}{2}\cdot\frac{1}{6}\right)\left(x - \frac{\pi}{4}\right)^3$$

$$\approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$$