

Exam 3 Calc 1 3/31/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. If $f(x) = 9x - x^2$, what is the maximum value of $f(x)$?

2. If $g'(x) = 2x - e^x + \sin x$, find the most general antiderivative of $g'(x)$.

3. If a company's cost to produce x widgets is given by $C(x) = 5000 + 200x + 2x^2$, what are

a. The company's average cost at a production level of 100 widgets?

b. The company's marginal cost at a production level of 100 widgets?

4. Find the intervals of concavity and inflection point(s) of $f(x) = x e^{-x}$.

5. Evaluate $\lim_{x \rightarrow 0^+} x \ln x$.

6. Find the absolute maximum and minimum values of $f(t) = t\sqrt{5-t^2}$ on the interval $[0,2]$.

7. Use Newton's Method to approximate $\sqrt[3]{5}$. Start with $x_1 = 2$, and compute x_2 and x_3 to the nearest thousandth.

8. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this calculus stuff is so weird. We had this question on our test yesterday, and it was like, the answer was given, but you had to say *why*. I mean, I always thought it was the answer if the book said it was, right? So it was like, why is $\arcsin(x/a)$ the antiderivative for $\frac{1}{\sqrt{a^2 - x^2}}$? Which totally confuses me, because that was on the homework and someone asked about it in our discussion section, and the TA just said that's what it was. So isn't it totally unfair that we're supposed to know *why*?"

Explain clearly to Bunny how you know the antiderivative of $\frac{1}{\sqrt{a^2 - x^2}}$ is $\arcsin(x/a)$.

9. The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources, one three times as strong as the other, are placed 10 ft apart, **write an equation** whose solution will give the location where an object could be placed on the line between the sources so as to receive the least illumination. [Stewart 5th, p. 338].

10. For what values of the constant b does the function $f(x) = 2x^3 + bx^2 + 10x - 7$ have both a local maximum and local minimum point?

Extra Credit (5 points possible): [Stewart 5th, p. 365] If x , y , and z are positive numbers, show that

$$\frac{(x^2 + 1)(y^2 + 1)(z^2 + 1)}{xyz} \geq 8. \text{ [Hint: Warm up by thinking about } \frac{x^2 + 1}{x} \text{ .]}$$