Exam 4 Calc 1 2/21/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Suppose that the following table of values gives a car's speed as it accelerates after a light turns green, then brakes to a stop at the next intersection. Based on this data, give a lower approximation for the distance the car traveled between the two intersections.

t (seconds)	0	5	10	15
speed (feet/second)	5	40	50	10

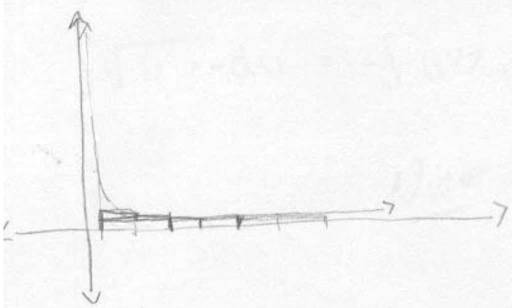
$5\frac{ft}{s}\cdot 5s + 40\frac{ft}{s}\cdot 5s + 10\frac{ft}{s}\cdot 5s = 275$ feet

Good

2. Use a right-hand sum with three equal subintervals to approximate the value of $\int_1^7 \frac{1}{x}\,dx$ to the nearest hundredth.

$\Delta n = 6/3 = 2$

* no discontinuity *



$R(x) = 2f(3) + 2f(5) + 2f(7)$

$= 2(1/3) + 2(\frac{1}{5}) + 2(1/7)$

$\approx \boxed{1.35}$

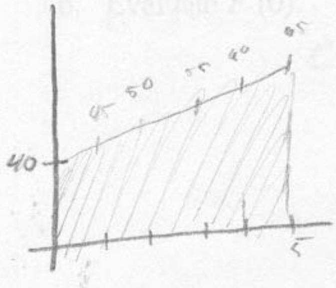
Great!

3. Evaluate $\int_1^7 \frac{1}{x}\,dx$ exactly.

$\int_1^7 \frac{1}{x}\,dx = \ln|x| \Big|_1^7 = \ln|7| - \ln|1| = \boxed{\ln 7}$

Excellent

4. If a company's expected profits t years from now, in millions of dollars per year, are given by $p(t) = 40 + 5t$, what total profit can the company expect over the next five years?



$$\int_0^5 40 + 5t$$

$$\left[40t + \frac{5}{2}t^2\right]_0^5$$

$$(200 + 62.5) - (0 + 0)$$

$$262.5$$

Great

5. Evaluate $\int \sqrt{4-t}\, dt$.

$$\int \sqrt{4-t}\ dt$$

$$u = 4-t$$

$$\frac{du}{dt} = -1$$

$$du = -dx$$

$$dt = -du$$

$$-\int \sqrt{u}$$

$$-1 \cdot \frac{2}{3}u^{3/2} + C$$

$$-\frac{2}{3}(4-t)^{3/2} + C$$

Nice!

6. Let $F(x) = \int_0^x \cos(\theta^3)\,d\theta$

Any integral from a to a will be zero!

a. Evaluate $F(0)$.

$F(0) = \int_0^0 \cos(\theta^3)\,d\theta = 0$

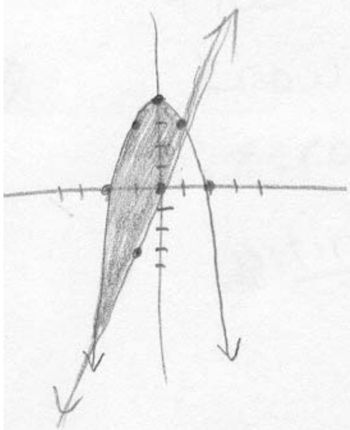
b. Evaluate $F'(0)$.

$F'(x) = \cos(x^3)$

By F.T.C.!

$F'(0) = \cos(0^3) = 1$

7. Find the area bounded between $y = 4 - x^2$ and $y = 3x$.



$4 - x^2 = 3x$

$0 = x^2 + 3x - 4$

$0 = (x+4)(x-1)$

$x = -4, 1$

$\int_{-4}^{1} \left((4-x^2) - 3x\right) dx$

$= \int_{-4}^{1} (4 - 3x - x^2)\,dx$

$= \left[4x - \frac{3}{2}x^2 - \frac{x^3}{3}\right]_{-4}^{1}$

$= \left(4(1) - \frac{3}{2}(1)^2 - \frac{(1)^3}{3}\right) - \left(4(-4) - \frac{3}{2}(-4)^2 - \frac{(-4)^3}{3}\right)$

$= 4 - \frac{3}{2} - \frac{1}{3} + 16 + 24 - \frac{64}{3}$

$= \boxed{\frac{125}{6}}$

Excellent

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, I think they just try to make this calculus stuff harder than it really is. Our TA was making this big deal about how you can only do definite integrals if they're continuous or something, but that's crap. There was one on our test where it wasn't continuous, and I did it anyway, and the answer I got was right there on the multiple choice list. Why would it be such a big deal if it wasn't continuous?"

Explain clearly to Biff why it indeed does matter if the integrand is continuous, or when it doesn't matter.

Biff, I think maybe you didn't completely understand what your TA was saying. It's certainly possible to do some definite integrals where the integrand isn't continuous, and your textbook probably has some problems like that, either with functions with piecewise-defined formulas or pictures of graphs that are discontinuous. Lots of them can be done just fine by dealing with each piece separately.

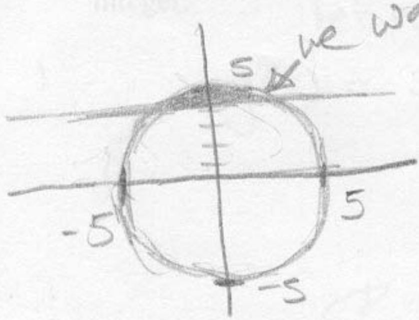
But what you can't do with a discontinuous function is find the definite integral using the Fundamental Theorem of Calculus. If the discontinuity occurs in the interval you integrate on, Bad Things can happen. One example is $\int_{-2}^{3} \frac{1}{x^2}\,dx$, where if you don't realize that the usual way doesn't apply (because of the asymptote at $x=0$), you'd say:

$$\int_{-2}^{3} x^{-2}\,dx = -x^{-1}\Big|_{-2}^{3} = -\frac{1}{3} - \frac{-1}{-2} = -\frac{5}{6}$$

So you'd be getting a negative "area" for a region that's entirely above the x-axis, which you should realize is wrong.

So Biff, the bottom line is that you might still be able to do an integral when the function involved isn't continuous, you just can't do it the usual way using the Fundamental Theorem of Calculus.

9. **Set up** an integral for the area inside the circle $x^2 + y^2 = 25$, but above the line $y = 4$.



$$y^2 = 25 - x^2$$

$$y = \sqrt{25 - x^2}$$

$$y = 4$$

$$\sqrt{25-x^2} = 4$$

$$\sqrt{25-x^2} - 4 = 0$$

$$25 - x^2 - 16 = 0$$

$$9 - x^2 = 0$$

$$(x+3)(x-3) = 0$$

$$x = 3 \quad x = -3$$

$$\int_{-3}^{3} \left(\sqrt{25-x^2}\right) - 4 \, dx$$

$$\int_{-3}^{3} \left[(25-x^2)^{1/2} - 4\right] dx$$

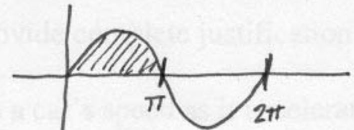
or since the graph is symmetrical you could do (if you're lazy)

$$2\int_{0}^{3} \left[(25-x^2)^{1/2} - 4\right] dx$$

Well done

10. Find the area of one arch of a function of the form $g(x) = \sin(nx)$, where n is some positive integer.

Let's warm up with $n = 1$:



$$\int_0^{\pi} \sin x\, dx$$
$$= -\cos x \Big|_0^{\pi}$$
$$= -\cos\pi - -\cos 0$$
$$= 1 + 1$$
$$= 2$$

Now with a general n, I need limits. Where does $y = \sin(nx)$ cross the x-axis? Definitely when $x = 0$, but then next when $nx = \pi$ (since $\sin\pi = 0$), so where $x = \frac{\pi}{n}$.

So Area $= \displaystyle\int_0^{\pi/n} \sin(nx)\, dx$

let $u = nx$

$\frac{du}{dx} = n$

$\frac{du}{n} = dx$

$$= \int_{x=0}^{x=\frac{\pi}{n}} \sin u \cdot \frac{du}{n}$$

$$= \frac{1}{n} \cdot -\cos u \Big|_{x=0}^{x=\frac{\pi}{n}}$$

$$= \frac{-1}{n} \cdot \cos(nx) \Big|_0^{\pi/n}$$

$$= \frac{-1}{n}\cos\left(n \cdot \frac{\pi}{n}\right) - \frac{-1}{n}\cos(n \cdot 0)$$

$$= \frac{-1}{n} \cdot \cos\pi + \frac{1}{n} \cdot \cos 0$$

$$= \frac{-1}{n} \cdot -1 + \frac{1}{n} \cdot 1 = \frac{2}{n}$$