Exam 1 Differential Equations 2/6/04

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Which of the following are differential equations? Circle all that are.

a) $\mathbf{X} = (\mathbf{A} - \lambda\mathbf{I})$

b) $y' = 3yt$

c) $\dfrac{dy}{dt} + f(y)g(t) - 3 \cdot \dfrac{y}{30+t}$

d) All men are mortal, and Socrates is a man, therefore all men are Socrates.

e) $y = Ae^{-3t} - B$

2. Sketch the phase line for the differential equation $\dfrac{dP}{dt} = 0.3P\left(1 - \dfrac{P}{2000}\right)$, and identify any equilibrium point(s) as sources, sinks, or nodes.

3. The differential equation for the temperature of a differential equations book placed in a 2000° furnace is $\frac{dH}{dt} = k(2000 - H)$, at least up until around where H(t) = 451°. This equation has general solutions of the form $H(t) = 2000 - Ae^{-kt}$. If a book has a temperature of 70° when first thrown into the furnace, and two minutes later has heated up to 200°, find a particular solution (with values accurate to two significant figures) fitting this situation.

4. Jon's water conditioner has been sabotaged so that it emits water with 5 grams of pink ink in every gallon of water. If Jon is running water at a rate of 3 gallons per minute into his 10 gallon sink, and the sink is half full of clean water before the ink solution begins to come out, write a differential equation for the amount of pink ink in the sink t minutes after the ink starts to come out, and an initial condition representing the situation.

5. Find a general solution to the differential equation $\frac{dy}{dt} = \beta y - \alpha$.

6. Show that if $f(x)$ is an integrable function, $g(x)$ is a differentiable function, and $u = g(x)$, then
$$\int f(g(x)) \cdot g'(x)\, dx = \int f(u)\, du$$

7. Find a general solution to the differential equation $\frac{dy}{dt} = e^{2t} + 5y$.

8. **Sketch the bifurcation diagram** for the differential equation $\frac{dy}{dt} = \left(y^2 - a \right)\left(y^2 - 4 \right)$. Include direction arrows on the phase lines and make clear the exact α values where bifurcations occur.

9. Solve the initial value problem $t\dfrac{dy}{dt} = y + \left(t^2 - y^2\right)^{1/2}$, $y(1) = 0$ by using the substitution $u = \dfrac{y}{t}$.

10. Somebody tells you that if two functions are each solutions to a differential equation, then their product must also be a solution to that differential equation. Are they right? Why or why not?