## Examlet 3 Foundations of Advanced Math 3/24/06

Each problem is worth 10 points. Appropriate justification is required for full credit.

- 1. For the following, complete proofs that properties **do** hold are not necessary but justification when they **do not** hold is expected. Be explicit about the domains and codomains:
  - a) Give an example of a function which is neither one-to-one nor onto.

b) Give an example of a function which is both one-to-one and onto.

c) Give an example of a function which is one-to-one but not onto.

2. Prove that any function  $f: \mathbb{R} \to \mathbb{R}$  of the form f(x) = mx + b, for constant real numbers *m* and *b* (with  $m \neq 0$ ), is a surjection.

3. Let  $f: A \rightarrow B$ ,  $g: B \rightarrow C$ . Prove that if f and g are injective, then  $h = g \circ f$  is injective.

4. Suppose that A is a denumerable set, and  $x \in A$ . Show that  $A - \{x\}$  is denumerable.

5. Let *f*: ℝ → ℝ be a decreasing function.
a) What can be said about *f* ∘ *f* ?

b) Suppose that *f* is composed with itself *n* times, and denote the resulting function  $\bigcap_{i=1}^{n} f(x)$ . What can be said about  $\bigcap_{i=1}^{n} f$ ?

Extra Credit [2 points possible]: Show that the rational numbers are denumerable.