Problem Set 4 Foundations Due 2/20/2006

Each problem is worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

- Let A be a set of real numbers. We say that A is bounded iff $\exists N \in \mathbb{R}$ ($\forall x \in A$, $|x| \leq N$).

- The set {0, 1, 2, 3} is bounded.
- The sets $\mathbb{N}$ and $\mathbb{R}$ are not bounded.
- The set $\{x \in \mathbb{R} \mid x < 0\}$ is not bounded.

1. Any set containing exactly n elements, for some $n \in \mathbb{N}$, is bounded.

- Let A and B be sets of real numbers. Define $A \oplus B = \{z \in \mathbb{R} | z = x + y$ for some $x \in A, y \in B\}$.
- Let A and B be sets of real numbers. Define $A \otimes B = \{z \in \mathbb{R} | z = x \times y$ for some $x \in A, y \in B\}$.

2. If A and B are bounded sets of real numbers, then $A \oplus B$ is bounded.

- Let A be a set of real numbers and n a natural number. We say that A is n-dense iff $\forall x \in A$, $\exists y \in A$ such that $0 < |x - y| \leq n$.

- The set {1, 2, 3, 4} is 1-dense.
- The set {1, 2, 5} is 3-dense but not 2-dense or 1-dense.
- The set $\mathbb{N}$ is 1-dense, since $\forall n \in \mathbb{N}$, we have $n + 1 \in \mathbb{N}$ with $|n - (n + 1)| = 1$.
- The set $\{2^n | n \in \mathbb{N}\}$ is not n-dense for any n.
- The set {5} is not n-dense for any n.

3. If A and B are n-dense sets, then $A \oplus B$ is n-dense.

4. If A and B are n-dense sets, then $A \otimes B$ is n-dense.

5. If A and B are sets of real numbers and $A \oplus B$ is n-dense, then A and B are both n-dense.