Examlet 1a Foundations of Advanced Math 2/2/07

. a) State the definition of an odd integer.

An odd integer is an integer that can be expressed in the form of $n = 2m+1$ by definition for the integers n and m.

Yes

b) Is the statement "$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x \cdot y = 3)$" true or false? Support your answer.

The statement is false.

Proof. Let $x=0$. 0 multiplied by and other real number will always be 0. $\square$

Good

3. Show that if n is an integer for which n^3 is odd, then n is odd.

Well, s'pose n^3 were odd, and let's assume n is even in hope of obtaining a contradiction. Then $n = 2p$ for some $p \in \mathbb{Z}$, and

$$n^3 = (2p)^3 = 8p^3 = 2(4p^3).$$

But since p was an integer, $4p^3$ is also an integer, and thus n^3 is even, a contradiction arising from our assumption that n was even. Then instead n must be odd, as desired. $\square$

2. a) Make a truth table for the statement P ∧ Q.

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

Good

b) Determine whether the propositional (P ∨ Q) ⇒ R is equivalent to (P ⇒ R) ∨ (Q ⇒ R). *

P	Q	R	P∨Q	(P∨Q) ⇒ R *	P ⇒ R	Q ⇒ R	(P⇒R)∨(Q⇒R)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

As the Two colums with * are not equal the two propositionals are not equivalent.

Great

4. Show that if x is rational and y is irrational, then $x + y$ is irrational.

Let $x = \frac{a}{b}$ by def. $a, b \in \mathbb{Z}$

Let's assume that $x+y$ is rational,

Then the sum of $x+y$, call it m can

be defined as

$$m = \frac{n}{o} \qquad n, o \in \mathbb{Z}$$

Then,

$$x + y = m$$

$$\frac{a}{b} + y = \frac{n}{o}$$

Then solving for y

$$y = \frac{n}{o} - \frac{a}{b}$$

Nice!

$$y = \frac{nb - ao}{bo}$$

Since $a, b, n, + o$ are all defined as integers, by closure, the value of y can be written as

$$y = \frac{f}{g} \qquad f, g \in \mathbb{Z}$$

+ by def. y is now a rational number which is a contradiction to the initial statement. The sum must be irrational $\square$

5. Show that $(\forall n \in \mathbb{N}, n \geq 4)(n! > 2^n)$.

Proof: Let's use induction!

First consider when $n=4$. Then,

$$4! \overset{?}{>} 2^4$$

$$4 \cdot 3 \cdot 2 \cdot 1 \overset{?}{>} 2^4$$

$$24 > 16$$ ✓ This is true.

Suppose this statement holds true when $n=k$. Then,

$$k! > 2^k$$

Next, consider when $n=k+1$. Then,

Well done!

$$(k+1)! > 2^{k+1}$$

$$\underline{\underline{k!}}(k+1) > \underline{\underline{2^k}} \cdot 2$$

Well, we know ~~that~~ from our original statement that $k!$ is greater than 2^k. In addition to this, $n \geq 4$, so $k+1 \geq 4$, or $k+1 \geq 2^2$.

So because $k! > 2^k$ and $2^2 > 2$, then

$$(k+1)! > 2^{k+1}.$$

Because this statement holds true for $n=4$, and when $n=k$ then $n=k+1$ is also true, then by mathematical induction, this statement ~~holds~~ is true. □