Examlet 2b **Foundations of Advanced Math** **2/23/07**

1. a) If A and B are sets, state the definition of $A \cup B$.

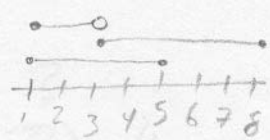
$$A \cup B \equiv \{x \mid (x \in A) \vee (x \in B)\}$$

Good

b) Let $C = \{1,2,3\}$ and $D = \{3,4,5\}$. What is $C \cap D$?

$$C \cap D = \{3\}$$

c) Let $E = [1,5]$ and $F = [3,8]$. What is $E - F$?



$$E - F = [1,3)$$

2. a) Suppose $A_n = [1/n, n+3]$ for all $n \in \mathbb{N}$. What is $\bigcup_{n \in \mathbb{N}} A_n$?

$A_1 = \left[\frac{1}{1}, 1+3\right] = [1, 4]$

$A_2 = \left[\frac{1}{2}, 2+3\right] = \left[\frac{1}{2}, 5\right]$

$A_3 = \left[\frac{1}{3}, 3+3\right] = \left[\frac{1}{3}, 6\right]$

$\underline{\mathbb{R}^+}$

b) Let $A_i = [1/n, n+3]$ for all $n \in \mathbb{N}$ as in part a. What is $\bigcap_{n \in \mathbb{N}} A_n$?

$\underline{[1, 4]}$

c) Let $B = \{a, b, c\}$ and $C = \{1, 2\}$. What is $B \times C$?

$\underline{\{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}}$

Great

3. a) Prove or give a counterexample: If $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$, then $a + c < b + d$.

$a < b$ $\quad$ $c < d$ $\quad$ what we are given

add 'a' to both sides $\quad$ $a+c < b+c$ $\quad$ $b+c < b+d$ $\quad$ add 'b' to both sides

$a+c < b+c < b+d$ $\quad$ combine to show relationship

Great

So it is true that $a+c < b+d$

b) Prove or give a counterexample: If $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$, then $a - c < b - d$.

Counter example

Let $a = 1$ $\quad$ $b = 2$ $\quad$ so $a < b$

Let $c = -50$ $\quad$ $d = 0$ $\quad$ so $c < d$

then $\quad a - c \not< b - d$

because $\quad 1 - (-50) \not< 2 - 0$

$51 \not< 2$

Good

So it is proved false by counter example

4. Let $\{A_i \mid i \in I\}$ be an indexed family of sets, and let B be any set, all subsets of some universal set. Show that $B \cup \bigcup_{i \in I} A_i = \bigcup_{i \in I} (B \cup A_i)$.

$D = B \cup \bigcup_{i \in I} A_i \qquad E = \bigcup_{i \in I} (B \cup A_i)$

Let $x \in D$, then $x \in B \cup \bigcup_{i \in I} A_i$. So $x \in B$ or $x \in \bigcup_{i \in I} A_i$.

If $x \in \bigcup_{i \in I} A_i$, then $x \in A_i$ for some $i \in I$. So we have $x \in B$ or $x \in A_i$ for some $i \in I$, which is the same as $x \in B \cup A_i$ for some $i \in I$. This leads to $x \in \bigcup_{i \in I} (B \cup A_i)$.

Thus $D \subseteq E$.

Let $x \in E$, then $x \in \bigcup_{i \in I} (B \cup A_i)$. Then $x \in B \cup A_i$ for some $i \in I$. Then $x \in B$ or $x \in A_i$ for some $i \in I$. If $x \in A_i$ for some $i \in I$, then $x \in \bigcup_{i \in I} A_i$. Thus $x \in B$ or $x \in \bigcup_{i \in I} A_i$. Then $x \in B \cup \bigcup_{i \in I} A_i$. Thus $E \subseteq D$.

Since $D \subseteq E$ and $E \subseteq D$, it is true that $D = E$. □

Nice!

5. Let A, B, C, and D be sets. Show that if $A \subseteq B \cap C$, then $A - D \subseteq B$.

Proof: Well, take $x \in A - D$, so $x \in A$ and $x \notin D$. Then since $A \subseteq B \cap C$, $x \in A \Rightarrow x \in B \cap C$. So $x \in B$ and $x \in C$. But this means $x \in A - D$ always implies $x \in B$, so $A - D \subseteq B$ as desired. □