## Examlet 4a Foundations of Advanced Math 4/13/07

1. a) Let  $A = \{5, 6, 7, 8, 9\}$  and  $R_a = \{(5,5), (5,9), (6,6), (6,7), (7,6), (7,7), (8,8), (9,5), (9,9)\}$ . What partition  $P_a$  corresponds to the relation  $R_a$  on  $P_a$ ?

b) Let *A* be as above, and let  $P_b = \{\{5\}, \{6,7,8\}, \{9\}\}$ . What relation corresponds to the partition  $P_b$ ?

2. a) Give an example of a relation on the set  $\{1,2,3,4,5\}$  that is reflexive, symmetric, and transitive.

b) Give an example of a relation on the set {1,2,3,4,5} that is reflexive, not symmetric and not transitive.

3. Consider the relation  $\triangleleft$  on  $\mathbb{R}$  defined by  $x \triangleleft y \Leftrightarrow x \le 2y$ . Determine whether  $\triangleleft$  is reflexive, symmetric, or transitive, and justify your conclusions clearly.

4. Let  $\mathcal{F}$  be a partition of a set A. Define a relation R on A by

$$(a,b) \in R \Leftrightarrow (\exists X \in \mathcal{F}) [a, b \in X]$$

Show that R is an equivalence relation on A.

5. a) Regarding the function  $f: A \rightarrow B$  as a subset of  $A \times B$ , write the definition of  $f^{-1}$ .

b) Let *A* be some set. Write the identity function  $i: A \rightarrow A$  as a relation on  $A \times A$ .