## Examlet 4a Foundations of Advanced Math 4/13/07

1. a) Let $A=\{5,6,7,8,9\}$ and $R_{a}=\{(5,5),(5,9),(6,6),(6,7),(7,6),(7,7),(8,8),(9,5),(9,9)\}$. What partition $P_{a}$ corresponds to the relation $R_{a}$ on $P_{a}$ ?
b) Let $A$ be as above, and let $P_{b}=\{\{5\},\{6,7,8\},\{9\}\}$. What relation corresponds to the partition $P_{b}$ ?
2. a) Give an example of a relation on the set $\{1,2,3,4,5\}$ that is reflexive, symmetric, and transitive.
b) Give an example of a relation on the set $\{1,2,3,4,5\}$ that is reflexive, not symmetric and not transitive.
3. Consider the relation $⿶$ on $\mathbb{R}$ defined by $x \triangleleft y \leftrightarrow x \leq 2 y$. Determine whether $⿶$ is reflexive, symmetric, or transitive, and justify your conclusions clearly.
4. Let $\mathcal{F}$ be a partition of a set $A$. Define a relation $R$ on $A$ by

$$
(a, b) \in R \Leftrightarrow(\exists X \in \mathcal{F})[a, b \in X]
$$

Show that $R$ is an equivalence relation on $A$.
5. a) Regarding the function $f: A \rightarrow B$ as a subset of $A \times B$, write the definition of $f^{-1}$.
b) Let $A$ be some set. Write the identity function $i: A \rightarrow A$ as a relation on $A \times A$.

