Problem Set 10   Foundations   Due 4/25/2007

Each problem is worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

▸   Given a set A, define $S(A)$ to be the set $A \cup \{A\}$.

1. Write $S(\varnothing)$, $S(S(\varnothing))$, and $S(S(S(\varnothing)))$ explicitly. How many elements does each of these have?

2. Show that for any set A, $S(A) \neq \varnothing$.

3. Define a relation $\approx$ on the collection of all sets by $\approx \, = \{(A,B) \mid \exists f{:}A \rightarrow B \text{ a bijection}\}$. Show that $\approx$ is an equivalence relation.

4. Regarding 2 as $S(1)$, 1 as $S(0)$, and 0 as $\varnothing$, and using the definition of addition in a Peano system (with $S(x)$ identified with x'), show that $2 + 2 = 4$. [It may be convenient to also regard $S(2)$ as 3, and $S(3)$ as 4.]