Problem Set 3   Foundations   Due 2/9/2007

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Critique the following "proof":

   Proposition: If n is an integer for which n^2 is throdd, then n is throdd.

   Proof: Well, let n be throdd, so $n = 3m + 1$ for some $m \in \mathbf{Z}$. Then

   $$n^2 = (3m + 1)^2 = 9m^2 + 6m + 1 = 3(3m^2 + 2m) + 1$$

   which is throdd since $3m^2 + 2m$ is an integer. ❑

2. Give a set-theoretic proof of part (1) of Theorem 3.2.1.

3. Give a set-theoretic proof of part (5) of Theorem 3.2.1.

4. Give a set-theoretic proof of (10)(c) in §3.2.

5. Give a set-theoretic proof of (11)(e) in §3.2.

6. Give an algebraic proof of the identity
   $$(A \cap B) \cup (C \cap D) = (A \cup C) \cap (A \cup D) \cap (B \cup C) \cap (B \cup D).$$