Problem Set 5 Foundations Due 3/2/2007

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Give an example of a function from (0,1) onto [0,1], or explain why one cannot exist.

2. Give an example of a function from (0,1) onto $\mathbb{R}$, or explain why one cannot exist.

- A set $A \subseteq \mathbb{R}$ is **bounded** iff there exists some $M \in \mathbb{R}$ such that $|x| < M$ for all $x \in A$.

- The set $\{1,2,3\}$ is bounded because $4 \in \mathbb{R}$ satisfies $|1| < 4$, $|2| < 4$, and $|3| < 4$.
- The set $\mathbb{N}$ is not bounded, since for any $M \in \mathbb{R}$, there is a natural number larger than M.
- The set $[0,1]$ is bounded, since $7 \in \mathbb{R}$ satisfies $|x| < 7$ for all $x \in [0,1]$. Note that 7 is certainly not the only bound for this set – the definition doesn't require uniqueness.

3. If A and B are bounded sets, then $A \cup B$ is a bounded set.

4. If A and B are bounded sets, then $A \cap B$ is a bounded set.

5. If $A \cap B$ is a bounded set, then A and B are bounded sets.

6. If $A \cup B$ is a bounded set, then A and B are bounded sets.