Exam 1 Calc 2 1/30/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Write an integral for the area of the region between the graph of $y = 9 - x^2$ and the x-axis.

2. Write an integral for the average value of the function $f(x) = x^3$ on the interval [0, 2].

3. Write an integral for the volume of the solid of revolution obtained by rotating the region between $y = 1/x$, $y = 0$, $x = 1$, and $x = 5$ around the x-axis.

4. Suppose that a spring with a natural length of 6 inches requires 200 pounds to hold it stretched to a length of 9 inches. How much work is required to stretch it from a length of 6 inches to a length of 12 inches?

5. Evaluate $\int_{e}^{e^2} \frac{dx}{x\sqrt{\ln x}}$.

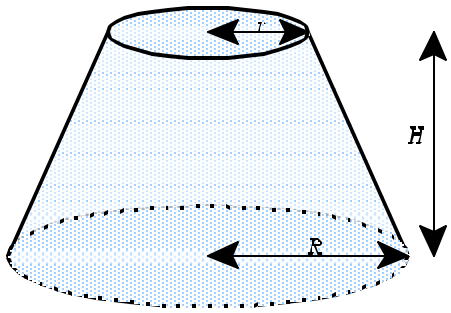
6. Write an integral for the volume of the solid of revolution obtained by rotating the region between $y = x^2$, $y = 0$, $x = 1$, and $x = 2$ about the axis $x = 4$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. We're doing these work problems in calc, and they're really complicated, but I think there's an easier way. If I can figure out the weight for all the water in a tank, and it's gotta get up 2 feet to go out some spout, I can just take the whole weight times 2 feet and that's work, right? So all this stuff with integrals is just to make it hard for us to weed us out from pre-engineering, right?"

   Help Biff by explaining to him why what he's proposing does or does not work.

8. Evaluate $\int x\sqrt{a+bx}\,dx$, where a and b are constants.

9. Write an integral for the volume of a frustum of a right circular cone with height h, lower base radius R, and top radius r. [Stewart]



10. Jon plans to build a water tower on top of his house *just in case*. The holding tank will be a sphere 5 feet in radius, with its bottom 25 feet above ground level. The water is pumped up from a well which draws water from 240 feet below ground level. Write an integral for the total amount of work required to fill the holding tank.

Extra Credit (5 points possible):

Show that when the region between $y = a\sqrt{x - ax^2}$ and the x-axis is rotated around the x-axis, the resulting volume is independent of the constant a. [Rogawski p. 415]