**Examlet 3 Foundations of Advanced Math 3/21/08**

1. a) State the definition of an increasing function $f\colon \mathbb{R} \to \mathbb{R}$.

   b) State the definition of an odd function $f\colon \mathbb{R} \to \mathbb{R}$.

2. Let $f\colon A \to B$ be invertible. Show that $f^{-1} \circ f = I_A$.

3. Let $f: A \to B$ and $g: B \to C$ be injective functions. Show that $g \circ f$ is injective.

4. a) Show that $\mathbb{Z}$ is denumerable.

b) Show that if $A$ is uncountable and $x$ is an object in $A$, then $A - \{x\}$ is uncountable.

5. Let $\{A_i \mid i \in \mathbb{N}\}$ be an indexed family of sets, and suppose that $A_i$ is bounded for every $i \in \mathbb{N}$. Let $Z_n = \{m \in \mathbb{N} \mid m \leq n\}$. Show that $\bigcup_{i \in Z_n} A_i$ is bounded for all $n \in \mathbb{N}$.