Problem Set 5 Foundations Due 2/29/2008

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. Suppose that A is a set of real numbers and there exists $x_0 \in A$ for which $|x - x_0| \le r$ holds for all $x \in A$. Is it necessarily true that for all $x, y \in A$, we will have $|x - y| \le r$?

2. Suppose that A is a set of real numbers and there exists $x_0 \in A$ for which $|x - x_0| \le r$ holds for all $x \in A$. Is it necessarily true that for all $x, y \in A$, we will have $|x - y| \le 2r$?

- ▸ A set $A \subseteq \mathbb{R}$ is **bounded** iff there exists some $M \in \mathbb{R}$ such that $|x| < M$ for all $x \in A$.

- ♦ The set $\{1,2,3\}$ is bounded because $4 \in \mathbb{R}$ satisfies $|1| < 4$, $|2| < 4$, and $|3| < 4$.
- ♦ The set $\mathbb{N}$ is not bounded, since for any $M \in \mathbb{R}$, there is a natural number larger than M.
- ♦ The set $[0,1]$ is bounded, since $7 \in \mathbb{R}$ satisfies $|x| < 7$ for all $x \in [0,1]$. Note that 7 is certainly not the only bound for this set – the definition doesn't require uniqueness.

3. If A and B are bounded sets, then $A \cup B$ is a bounded set.

4. If A and B are bounded sets, then $A \cap B$ is a bounded set.

5. If $A \cap B$ is a bounded set, then A and B are bounded sets.

6. If $A \cup B$ is a bounded set, then A and B are bounded sets.

7. Let $\{A_i \mid i \in I\}$ be an indexed family of sets. If A_i is a bounded set for all $i \in I$, then $\bigcup_{i \in I} A_i$ is a bounded set.

8. Let $\{A_i \mid i \in I\}$ be an indexed nonempty family of sets. If A_i is a bounded set for all $i \in I$, then $\bigcap_{i \in I} A_i$ is a bounded set.