Exam 1   Calc 2   2/6/2009

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Write an integral representing the area between $y = 0$ and $y = x^2$ between $x = 1$ and $x = 7$.

2. Write an integral representing the average value of the function $f(x) = \dfrac{\sin x}{x}$ on the interval [p/2, 3p/2].

3. Evaluate the integral $\int \sqrt{1+x^2}\ x^5\ dx$.

4. Write an integral for the volume of the solid generated by rotating the region bounded between the curves $y = x^2$ and $x = y^2$ around the axis $x = -5$.

5. If a spring has a natural length of 30cm, and 5J of work is required to stretch it to a length of 40cm, how much work would be required to stretch it from 30cm to 35cm?

6. An Olympic-size swimming pool is 2 meters deep, 25 meters wide, and 50 meters long. Set up an integral for the amount of work required to pump all of the water in a (completely full) Olympic-size pool out over the edge of the pool, while Michael Phelps sits nearby looking confused.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, this solid revolution stuff is way too hard for me. I heard these foreign guys in my class talk about it, though, and they was sayin' there was this way where, like, you just take the area of the stuff, like before it gets rotated, right? And you just take that times two pi and it gives you the volume. It figures they wouldn't tell us anything that easy, 'cause then everyone would pass the class, but now that I know I'm gonna blow through our exam tomorrow!"

Explain clearly to Biff why his proposed approach should or should not be depended on.

8. Consider the regions bounded between curves of the form $y = x - x^a$ and the x-axis for integer values of a which are greater than or equal to 2. If such a region is rotated around the y-axis, what is the volume of the region created?

9. The city of Ringville has a population density of 100,000 people per square mile at its center, which decreases steadily to 50,000 people per square mile at a distance of 5 miles from the center of the city. What can you say about the total population living within 5 miles of the center of Ringville?

10. A torus is the solid created when a circle is rotated around an axis outside the circle (so the shape of a donut, or an inner tube). Suppose a torus is formed by rotating the circle $(x - 3)^2 + y^2 = 1$ around the y-axis (where the scale is in feet), and that the circle is filled with water (which weighs 62.5 lbs per ft^3). Write an integral for the amount of work required to pump all of the water out of the top of the torus. [If you can't manage the full integral, at least give an intelligent approximation.]

Extra Credit (5 points possible):
Take a circle with radius 1. Next draw two more circles centered at the topmost and bottommost points of the first circle, and large enough to pass through the leftmost and rightmost points of the first circle. Finally draw two additional circles centered at the leftmost and rightmost points of the first circle, and large enough to pass through the topmost and bottommost points of the first circle. What is the area of the region common to all five of these circles?