Exam 3 Calc 2 4/3/2009

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) Write out the first three terms in the sequence $\left\{\frac{1}{n^2}\right\}$.

b) Find the first three partial sums of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

2. Find the sum of the series $6-2+\frac{2}{3}-\frac{2}{9}+\ldots$

3. For what values of r does the function $y = e^{rx}$ satisfy the differential equation $2y'' + y' - y = 0$?

4. Sketch a good graph of the equation $\frac{x^2}{25} + \frac{(y-3)^2}{9} = 1$.

5. Find an equation for the line tangent to the curve with parametric equations $x(t) = t^4 + 1$, $y(t) = t^3 + t$ at the point where $t = -2$.

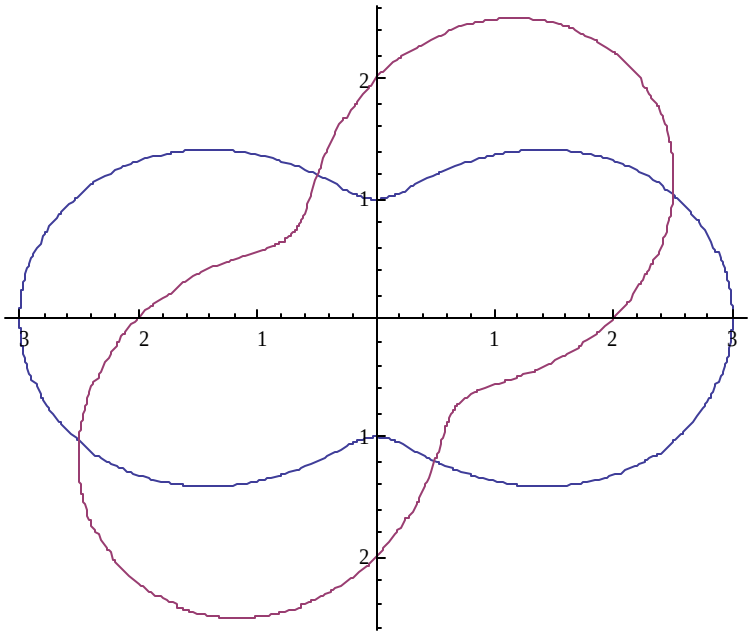
6. Suppose that an 80° cup of very bad soy half-decaf latte is left sitting in a 20° room because nobody wants to drink it, and that initially the liquid cools at a rate of 1° each minute. Use Euler's Method with a step size of 5 to approximate the temperature of the latte after 10 minutes.

7. Bunny is a calculus student at Enormous State University, and she has a question. Bunny says "Ohmygod, this is so amazing. I was reading in our Calculus book, like it's the same one you use, right? And there was this example where they, like, showed that the circumference of a circle with radius 1 is 4p instead of 2p! That's so amazing! I thought from Geometry in high school that it was always 2p times the radius, but I didn't know it could be different if you wrote the equation for the circle this parametric way. So, like, I wonder how many other circumferences that circle can have if you take even more math classes?"

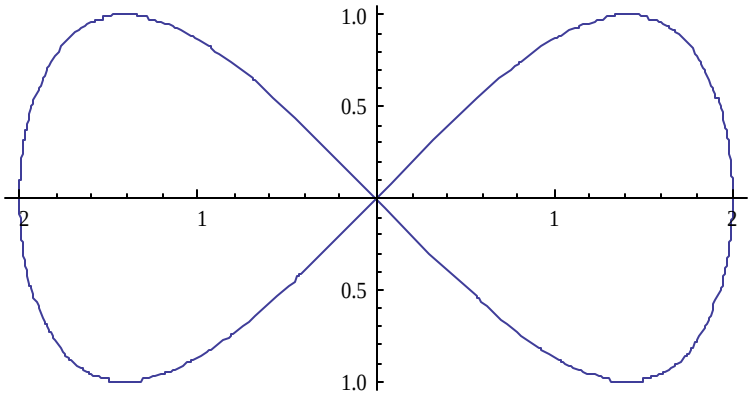
   Help Bunny by explaining what's going on.

8. Suppose that the performance, $P(t)$, of students given a length of time t to learn material is modeled by the differential equation $\frac{dP}{dt} = k\left(M - P\right)$ where M and k are positive constants. Find a solution $P(t)$ to this differential equation. What happens to $P(t)$ over the long run?

9. The graphs of $r = 2 + \sin 2\theta$ and $r = 2 + \cos 2\theta$ are shown below. Set up an integral (or integrals) for the area of the region inside both curves.



10. The curve with parametric equations $x(t) = 2\cos t$, $y(t) = \sin 2t$ is shown below. Find the area bounded by this curve. [Hint: It may be useful to use the trig identity $\sin 2x = 2\sin x \cos x$.]



Extra Credit (5 points possible):

Suppose a sequence is defined by letting $a_1 = 0$ and then $a_{n+1} = \sqrt{1 + a_n}$. Will it converge? How do you know?