

**The Geometric Series Test:** If a series is of the form  $\sum_{n=1}^{\infty} a \cdot r^{n-1}$ , then the series converges

$\left( \text{to } \frac{a}{1-r} \right)$  if and only if  $|r| < 1$

**The Integral Test:** Suppose  $f(x)$  is a continuous, positive, decreasing function on  $[c, \infty)$  for some  $c \geq 0$ , with  $a_n = f(n)$  for all  $n$ ,

- If  $\int_c^{\infty} f(x) dx$  converges, then  $Sa_n$  converges also.
- If  $\int_c^{\infty} f(x) dx$  diverges, then  $Sa_n$  diverges also.

**The Comparison Test:** If  $\sum a_n$  and  $\sum b_n$  are both series with their terms all positive, and

- $a_n \leq b_n$  with  $\sum b_n$  convergent, then  $\sum a_n$  converges also.
- $a_n \geq b_n$  with  $\sum b_n$  divergent, then  $\sum a_n$  diverges also.

**The Limit Comparison Test:** If  $\sum a_n$  and  $\sum b_n$  are both series with their terms all positive, and

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$$

for some finite, positive number  $L$ , then either both series converge or both series diverge.

**The Alternating Series Test:** If  $Sa_n$  is a series for which

- the signs alternate, i.e.  $a_n$  and  $a_{n+1}$  have opposite signs for all  $n$
- the sequence  $\{|a_n|\}$  tends to zero, i.e.  $\lim_{n \rightarrow \infty} |a_n| = 0$
- the sequence  $\{|a_n|\}$  is decreasing, i.e.  $|a_{n+1}| \leq |a_n|$  for all  $n$

then the series converges.

**The Ratio Test:** If  $Sa_n$  is a series for which

$$\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} = L$$

then

- if  $L < 1$  then the series converges absolutely.
- if  $L > 1$  (or if the limit diverges to  $+\infty$ ) then the series diverges.