Examlet 4 Foundations of Advanced Math 4/16/10

1. a) State the definition of a reflexive relation.

   b) Give an example of a relation on the set {1, 2, 3} which is symmetric but not transitive.

2. a) Suppose that ~ is an equivalence relation on the set $A = \{a, b, c, d, e\}$ and that $[a] = \{a, b, c\}$ and $[d] = \{d, e\}$. Write the partition P corresponding to ~.

   b) Suppose that P is the partition {{1}, {2, 4}, {3, 5}} of the set $A = \{1, 2, 3, 4, 5\}$. Find the relation ≈ corresponding to P.

3. Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \sim (c, d) \Leftrightarrow |a - c| + |b - d| \in \mathbb{Z}$. Determine whether R is reflexive, symmetric, or transitive, and support your conclusions well.

4. a) State the definition of a graph.

b) For the vertex set $V = \{a, b, c\}$, sketch all possible graphs (regarding a graph whose only edge connects a and b as different from one whose only edge connects b and c, for instance).

5. a) Regarding the function $f : A \to B$ as a subset of $A \times B$, write the definition of a surjection.

b) Regarding the function $f : A \to B$ as a subset of $A \times B$, write the definition of a bounded function.