Examlet 4 Foundations of Advanced Math 4/16/10

1. a) State the definition of a reflexive relation.

~~The~~ Given a set S, for all $s \in S$, s is related to s.

b) Give an example of a relation on the set $\{1, 2, 3\}$ which is symmetric but not transitive.

$\{(1,2),(2,1),(2,3),(3,2)\}$

Excellent

Not transitive because $1 \sim 2 \wedge 2 \sim 3$ but $1 \not\sim 3$.

2. a) Suppose that $\sim$ is an equivalence relation on the set $A = \{a, b, c, d, e\}$ and that $[a] = \{a, b, c\}$ and $[d] = \{d, e\}$. Write the partition $\mathcal{P}$ corresponding to $\sim$.

$P = \{\{a, b, c\}, \{d, e\}\}$

Good.

b) Suppose that $\mathcal{P}$ is the partition $\{\{1\}, \{2, 4\}, \{3, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$. Find the relation $\approx$ corresponding to $\mathcal{P}$.

$R = \{(1,1), (2,2), (4,4), (2,4), (4,2), (3,3), (5,5), (3,5), (5,3)\}$

Great

3. Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \sim (c, d) \Leftrightarrow |a - c| + |b - d| \in \mathbb{Z}$. Determine whether R is reflexive, symmetric, or transitive, and support your conclusions well.

Reflexive $m \sim m$

$m=(a,b)$

$|a-a|+|b-b|=0$

$0 \in \mathbb{Z}$ $\therefore$ $(a,b) \sim (a,b)$, $m \sim m$ and the relation is reflexive

Symetric $m \sim n \Rightarrow n \sim m$

$m=(a,b)$ $(a,b) \sim (c,d)$

$n=(c,d)$

It is known $|a-c|+|b-d| \in \mathbb{Z}$

Since $|a-c|=|c-a|$ and $|b-d|=|d-b|$ it is also true that $|c-a|+|d-b| \in \mathbb{Z}$ and so $(c,d) \sim (a,b)$ or $n \sim m$ and the relation is symetric

Nice!

Transitive If $m \sim n \wedge n \sim p$ then $m \sim p$

$m=(a,b)$

$n=(c,d)$

$p=(e,f)$

We know that $a,b,c,d,e,f \in \mathbb{Z}$ so consider $|a-e|+|b-f|$. An integer minus an integer is an integer and an integer plus an integer is also an integer so $|a-e|+|b-f|$ is equal to some integer therefore $m \sim p$, the relation is transitive

Side Note

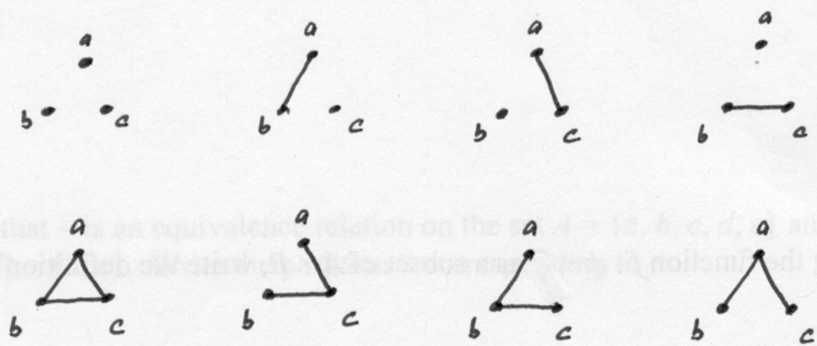
$\boxed{\text{So unless I'm mistaken } [(a,b)] = \mathbb{Z} \times \mathbb{Z}}$

Outstanding!

4. a) State the definition of a graph.

A graph is a set of vertices along with a set of edges, where each edge is a set containing exactly two vertices.

b) For the vertex set $V = \{a, b, c\}$, sketch all possible graphs (regarding a graph whose only edge connects a and b as different from one whose only edge connects b and c, for instance).



5. a) Regarding the function $f : A \to B$ as a subset of $A \times B$, write the definition of a surjection.

$\forall b \in B, \exists a \in A$ such that $(a,b) \in f$

Good

b) Regarding the function $f : A \to B$ as a subset of $A \times B$, write the definition of a bounded function.

$\exists$ some $m \in \mathbb{R}$ where $|b| \leq m$, $\forall (a,b) \in f$.

Great.