Problem Set 8   Foundations   Due 4/2/2010

The instructors will select four of these problems to grade, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

For each of the relations specified below:
a) Pick an element t of the set in question and find three other elements of the set which are related to it.
b) For your element t from part a, find three other elements of the set which are not related to it.
c) Determine whether the relation is reflexive, symmetric, or transitive.

1. Let $\approx$ be the relation on $\mathbb{Z} \times \mathbb{N}^+$ defined by $(a, b) \sim (c, d)$ iff $ab = cd$.

2. Let $\sim$ be the relation on $\mathbb{Z} \times \mathbb{N}^+$ defined by $(a, b) \sim (c, d)$ iff $ad = bc$.

3. Let $\propto$ be the relation on $\mathbb{R} \times \mathbb{R}$ defined by $(a, b) \propto (c, d)$ iff $\sqrt{(a-c)^2 + (b-d)^2} \leq 1$.

4. Critique the following "proof".

   Proposition: If $f\colon A \to B$ is a function and $g\colon B \to C$ is a surjection, then $g \circ f$ is surjective.

   "Proof": Well, we know for any $a \in A$, $f(a) = b$, where $b \in B$. Now take an arbitrary $c \in C$, and we know since g is surjective there's $b \in B$ so $g(b) = c$. Then $g \circ f\,(a) = g(f(a)) = g(b) = c$, so for an arbitrarily chosen $c \in C$ we have an $a \in A$ such that $g \circ f\,(a) = c$, and therefore $g \circ f$ is surjective by definition. $\square$