**The Geometric Series Test**: If a series is of the form  $\sum_{n=1}^{\infty} a \cdot r^{n-1}$ , then the series converges

$$\left(\text{to } \frac{a}{1-r}\right)$$
 if and only if  $|r| < 1$ 

**The Integral Test**: Suppose f(x) is a continuous, positive, decreasing function on  $[c, \infty)$  for some  $c \ge 0$ , with  $a_n = f(n)$  for all n,

- If  $\int_{c}^{\infty} f(x) dx$  converges, then  $\sum a_n$  converges also.
- If  $\int_{c}^{\infty} f(x) dx$  diverges, then  $\sum a_n$  diverges also.

**The Comparison Test:** If  $\Sigma a_n$  and  $\Sigma b_n$  are both series with their terms all positive, and

- $a_n \le b_n$  with  $\sum b_n$  convergent, then  $\sum a_n$  converges also.
- $a_n \ge b_n$  with  $\sum b_n$  divergent, then  $\sum a_n$  diverges also.

The Limit Comparison Test: If  $\Sigma a_n$  and  $\Sigma b_n$  are both series with their terms all positive, and

$$\lim_{n\to\infty}\frac{a_n}{b_n}=L$$

for some finite, positive number L, then either both series converge or both series diverge.

The Alternating Series Test: If  $\sum a_n$  is a series for which

- the signs alternate, i.e.  $a_n$  and  $a_{n+1}$  have opposite signs for all n
- the sequence  $\{|a_n|\}$  tends to zero, i.e.  $\lim_{n\to\infty} |a_n| = 0$
- the sequence  $\{|a_n|\}$  is decreasing, i.e.  $|a_{n+1}| \le |a_n|$  for all n

then the series converges.

**The Ratio Test**: If  $\sum a_n$  is a series for which

$$\lim_{n\to\infty}\frac{\left|a_{n+1}\right|}{\left|a_{n}\right|}=L$$

then

- if L < 1 then the series converges absolutely.
- if L > 1 (or if the limit diverges to  $+\infty$ ) then the series diverges.