Exam 3 Calc 2 4/6/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Give an example of a geometric series which converges, and find its sum.

2. a) Write a Taylor polynomial of degree 4 centered at $x = 0$ for $f(x) = e^x$.

   b) Write a Taylor polynomial of degree 4 centered at $x = 0$ for $f(x) = e^{3x}$.

3. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ converges or diverges.

4. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ converges or diverges.

5. Determine whether the series $\sum_{n=0}^{\infty} \frac{1}{n!+2}$ converges or diverges.

6. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$ converges or diverges.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. They make all this Calc stuff so hard, but it's just because they want everyone to drop so they don't have so much to grade. My buddy from class told me about how they don't actually tell you some of the best tests for those series things, because you'd just use the same one every time instead of having to try different ones. It's like a conspiracy. So like when they say if the ratio test says 1 then you don't know, well my buddy showed me actually if you do the ratio test on $1/n^2$, you get 1, and you know $1/n^2$ converges, so obviously really ones where you get 1 from the ratio test converge, and they just won't admit it."

   Help Biff by explaining clearly whether his conclusions are valid or not, and why.

8. Find the radius of convergence of the series $\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{4^k}$.

9. Use a Taylor series with at least 3 nonzero terms to approximate $\int_0^{0.2} \frac{\sin x}{x}\,dx$.

10. Use a Taylor series with at least 4 nonzero terms to approximate $\ln\left(\frac{3}{2}\right)$.

Extra Credit (5 points possible):

a) What is $e^{i\pi}$?

b) Use a Taylor series with at least 4 nonzero terms to approximate $\ln(5)$.