Exam 2a Differential Equations 3/16/12

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Find a solution to the system

$$\frac{dx}{dt} = 2x$$
$$\frac{dy}{dt} = 3y$$

2. Give an example of a partially decoupled system of differential equations.

3. Find all equilibrium points of the system

$$\frac{dR}{dt} = 2R - 1.2RF$$
$$\frac{dF}{dt} = -F + 0.9RF$$

4. Suppose that you know $x(t) = k_2 e^{2t} - \frac{k_1}{3} e^{-t}$, $y(t) = k_1 e^{-t}$ is a general solution to a system of differential equations. Find the solution satisfying the initial condition $\mathbf{Y}(0) = (x(0), y(0)) = (2, 1)$.

5. Consider the system $\begin{aligned} \frac{dR}{dt} &= 2R - 1.2RF \\ \frac{dF}{dt} &= -F + 0.9RF \end{aligned}$. Let $R(0) = 2$ and $F(0) = 1.5$, and use Euler's method with step size $\Delta t = 2$ to approximate $R(2)$ and $F(2)$.

6. Find a general solution to the system $\begin{aligned} \frac{dx}{dt} &= 3x + 2y \\ \frac{dy}{dt} &= 5x \end{aligned}$.

7. Let $y(t) = 5$. Find $\mathcal{L}[y]$, and note any necessary restrictions.

8. Compute the inverse Laplace transform $\mathcal{L}^{-1}\left[\frac{5}{(s-1)(s-2)}\right]$.

9. Suppose $a \geq 0$. Compute the Laplace transform of the function

$$r_a(t) = \begin{cases} 0 & \text{if } t < a \\ k(t-a) & \text{if } t \geq a \end{cases}.$$

10. Consider the second-order differential equation $y'' + \beta y' + 12y = 0$.

a) Let $\beta = 8$. Find a general solution to the equation.

b) Find a value for the parameter β for which $y(t) = e^{-3t}$ is a solution to this equation.