Problem Set 1 Differential Equations Due 2/3/12

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 5 points. For full credit indicate clearly how you reached your answer. All work must be legible and submitted on clean paper without ragged edges.

1. A fountain contains 100 ft^3 of water. At $t = 0$, 3 ounces of detergent are added. If the fountain's filtration system removes 20% of the detergent present per hour, produce a slope field representing the situation, predict the eventual behavior of the system, and find an exact solution for the amount of detergent remaining after t hours.

2. A fountain contains 100 ft^3 of water. At $t = 0$, a bar of soap is added, and slowly dissolves, releasing ½ ounce of detergent into the water each hour for 6 hours. If the fountain's filtration system removes 20% of the detergent present per hour, produce a slope field representing the situation, predict the behavior of the system, and find an exact solution for the amount of detergent remaining after t hours.

3. [Based on the ACM volume] A decent model for the velocity of a typical raindrop is
$$\frac{dv}{dt} = g - \frac{k}{D}v^2$$
where g is approximately 32 feet/second^2, D is the diameter of the raindrop in feet, and k is an empirically determined constant equal to 0.00046.

   For a raindrop with diameter 0.004 feet, describe the behavior of the velocity function clearly and find an exact solution to the differential equation assuming the raindrop starts at rest.

4. Suppose that a certain species of fish in a lake grows logistically with a carrying capacity of 5000 fish. It is experimentally determined that, in the absence of other factors, a stock of 1000 fish doubles after 1 year.

   If the population of the lake is allowed to grow to near the carrying capacity and then fishing is allowed at a rate of 1000 fish per year, predict the effect on the population. What about fishing 2000 fish per year? 3000?