Examlet 2 Foundations of Advanced Math 2/24/12

1. a) For any set A, $A - A = \varnothing$.

Well, take $x \in A - A$, so $x \in A \wedge x \notin A$, a contradiction, so $A - A \subseteq \phi$.

On the other hand, $\phi \subseteq A - A$ since we showed ϕ is a subset of any set.

Then since $A - A \subseteq \phi$ and $\phi \subseteq A - A$, we know $A - A = \phi$. □

b) For any sets A and B, if $A - B = \varnothing$, then $A = B$.

False. If $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then $A - B = \phi$, but $A \neq B$.

2. Let $C_n = (n - 2, n + 2)$ for each $n \in \mathbb{N}$.

a) What is $\bigcup_{i\in\{1,2,3\}} C_i$?

$\bigcup_{i\in\{1,2,3\}} C_i = C_1 \cup C_2 \cup C_3 = (1-2, 1+2) \cup (2-2, 2+2) \cup (3-2, 3+2) = (-1,3) \cup (0,4) \cup (1$

$\bigcup_{i\in\{1,2,3\}} C_i = \underline{(-1,5)}$

b) What is $\bigcap_{i\in\{1,2,3\}} C_i$?

$\bigcap_{i\in\{1,2,3\}} C_i = \underline{(1,3)}$

c) What is $\bigcup_{i\in\mathbb{N}} C_i$?

Excellent!

$\bigcup_{i\in\mathbb{N}} C_i = \underline{(-2, \infty)}$

d) What is $\bigcap_{i\in\mathbb{N}} C_i$?

$\bigcap_{i\in\mathbb{N}} C_i = \underline{\emptyset}$

3. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $\sqrt{ab} \leq \frac{a+b}{2}$.

Proof: In the supposition $a, b > 0$, and $a, b \in \mathbb{R}$, because we know $(a-b)^2 \geq 0$ because the square of a positive or negative number will be ≥ 0

$0 \leq (a-b)^2$

expand.

$0 \leq a^2 - 2ab + b^2$.

add $4ab$ to both sides of inequality.

$4ab \leq a^2 + 2ab + b^2$, comparison addition principle.

$4ab \leq (a+b)^2$

square root both sides.

$2\sqrt{a \cdot b} \leq a+b$, by a previous excercise in section 2.4.

$\frac{1}{2} \cdot 2\sqrt{a \cdot b} \leq a+b \cdot \frac{1}{2}$ comparison multiplication principle.

$\sqrt{a \cdot b} \leq \frac{a+b}{2}$. $\square$

Nice Job!

Scratch work.

$\cancel{2} \cdot \sqrt{a \cdot b} \leq \frac{a+b}{\cancel{2}} \cdot \cancel{2}$

$2 \sqrt{a \cdot b} \leq a+b$

$4 a \cdot b \leq (a+b)^2$

$4 a \cdot b \leq a^2 + 2ab + b^2$

$-4a \cdot b \quad -4 a \cdot b$

$0 \leq a^2 - 2ab + b^2$

$0 \leq (a-b)^2$

$a-b \quad a-b$

$a^2 - ba - ba +$

4. Suppose that $A_i \subseteq B_i$ for all $i \in I$. Then $\bigcap_{i\in I} A_i \subseteq \bigcup_{i\in I} B_i$.

Well, let's take $x \in \bigcap_{i\in I} A_i$. Thus, $x \in A_i$ for all $i \in I$ by definition of intersection. Since we are given $A_i \subseteq B_i$ for all $i \in I$, x must also be an element of B_i by definition of subsets. Since $x \in B_i$ for at least one $i \in I$, $x \in \bigcup_{i\in I} B_i$. Therefore, $\bigcap_{i\in I} A_i \subseteq \bigcup_{i\in I} B_i$. $\square$

Excellent!

5. a) State the Triangle Inequality.

$\forall x, y \in \mathbb{R} \quad |x+y| \leq |x| + |y|$

Good

b) For all $x, y, z \in \mathbb{R}$, $|x + y + z| \leq |x| + |y| + |z|$.

by definition of absolute value $-|x| \leq x \leq |x|$

$-|y| \leq y \leq |y|$

$-|z| \leq y \leq |z|$

by comparison addition principal all three inequalities can be added.

$-|x| - |y| - |z| \leq x+y+z \leq |x| + |y| + |z|$

$-(|x| + |y| + |z|) \leq x+y+z \leq |x| + |y| + |z|$

reversing what we have done using the definition of absolute value we can say

$|x+y+z| \leq |x| + |y| + |z|$. $\square$ boom

Nice!