

Problem Set 4 Foundations Due 2/18/2013

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

1. The sum of two rational numbers is rational.
2. The sum of two irrational numbers is irrational.
3. Consider the formula $1 + 2 + 3 + \dots + n = \frac{n^2 + n + 1}{2}$.
 - a) Write the formula in sigma notation.
 - b) Show that if this formula works for $n = k$, then it also must work for $n = k + 1$.
 - c) Explain why mathematical induction does **not** prove that this formula is true for all $n \in \mathbb{N}$.
4. For any integer n , the number $n^2 + n + 17$ is prime.
5. Prove that if A has n elements, then $\mathcal{P}(A)$ has 2^n elements. [Hint: Induction!]
6. For any sets A and B , $(A \cup B)' = A' \cap B'$.
7. For any sets A , B , and C , $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
8. Score at least 7 out of 8 on the Set Theory Gateway on WeBWorK, available via <http://webwork.coe.edu/webwork2/MTH-215/STGateway/> .

