## Examlet 2 Advanced Geometry 3/13/13

1. a) State the definition of an exterior angle of a triangle.
b) State the definition of $\sigma(\triangle A B C)$.
c) State the definition of a convex quadrilateral.
2. a) State the Exterior Angle Theorem.
b) State the Converse to the Isosceles Triangle Theorem.
c) State the Scalene Inequality.
d) State the Saccheri-Legendre Theorem.
e) State Hilbert's Parallel Postulate.
3. State five conditions that are equivalent to the Euclidean Parallel Postulate.
4. Prove that if $\ell$ and $\ell^{\prime}$ are two lines cut by a transversal $t$ in such a way that a pair of alternate interior angles is congruent, then $\ell$ is parallel to $\ell^{\prime}$.
5. Provide good justifications in the blanks below for the corresponding statements:

Angle-Side-Angle Theorem: If two angles and the included side of one triangle are congruent to the corresponding parts of a second triangle, then the two triangles are congruent.

Restatement: If $\triangle \mathrm{ABC}$ and $\triangle \mathrm{DEF}$ are two triangles such that $\angle \mathrm{CAB} \cong \angle \mathrm{FDE}, \overline{A B} \cong \overline{D E}$, and $\angle \mathrm{ABC} \cong \angle \mathrm{DEF}$, then $\triangle \mathrm{ABC} \cong \triangle \mathrm{DEF}$.

| Statement: | Reason: |
| :--- | :--- |
| Let $\triangle \mathrm{ABC}$ and $\Delta \mathrm{DEF}$ be two triangles such that <br> $\angle \mathrm{CAB} \cong \angle \mathrm{FDE}, \overline{A B} \cong \overrightarrow{D E}$, and $\angle \mathrm{ABC} \cong \angle \mathrm{DEF}$ |  |
| There exists a point $\mathrm{C}^{\prime}$ on $\overrightarrow{A C}$ such that <br> $A C^{\prime} \cong \overrightarrow{D F}$ |  |
| Now $\Delta \mathrm{ABC}^{\prime} \cong \Delta \mathrm{DEF}$ |  |
| and so $\angle \mathrm{ABC} \cong \angle \mathrm{DEF}$. | by hypothesis |
| Since $\angle \mathrm{ABC} \cong \angle \mathrm{DEF}$ |  |
| we can conclude $\angle \mathrm{ABC} \cong \angle \mathrm{ABC}{ }^{\prime}$. | Transitive Property |
| Hence $\overrightarrow{B C} \cong \overrightarrow{B C^{\prime}}$ |  |
| But $\overrightarrow{B C}$ can only intersect $\overleftrightarrow{A C}$ in at most one point | Theorem 3.1.7 |
| so $C=C^{\prime}$ and the proof is complete. | Because it's complete. |

